UNPUBLISHED PRELIMINARY DATA

FR-1036

7/p

N63 22510

CODE-1

SURFACE CHARACTERISTICS OF A PERTURBED SPHERE FROM RADAR RETURN SIGNAL

Final Report

NASA CR 51787

Contract No. NASw-612

OTS PRICE

XEROX $ 7.60 ph

MICROFILM $ 2.33 mf

For

National Aeronautics and Space Administration

Attn: Grants and Research Contracts

Code SC, Office of Space Sciences

Washington 25, D. C.

September 30, 1963

The Dikewood Corporation

4805 MENAUL BOULEVARD, N.E. ALBUQUERQUE, NEW MEXICO

Final Report

SURFACE CHARACTERISTICS OF A PERTURBED SPHERE FROM RADAR RETURN SIGNAL

F. H. Jean and
D. D. Babb Sept. 30, 1963

The Dikewood Corporation
4805 Menaul Boulevard NE
Albuquerque, New Mexico

(FR-1036)
(NASA Contract No. NASw-612)

Prepared for

National Aeronautics and Space Administration
Attn: Grants and Research Contracts
Code SC, Office of Space Sciences
Washington 25, D. C.

TABLE OF CONTENTS

ACKNOWLEDGMENT

Several members of the Dikewood Staff contributed to the ideas contained in this report. Particular recognition is due Donald L. Summers who performed the computer programming required in the course of the study.

ABSTRACT

22510

This report is a discussion of the problem of determining, through the analysis of radar return signals, the roughness of a spherical target such as a satellite or a planet. The following steps are made toward a solution of this problem: The impulse response of a perturbed sphere is obtained. Statistical relationships of a radar signal reflected from the sphere are developed. These relationships are employed to compute the ensemble average and autocorrelation of the return signal. The computed results obtained are compared with experimental data. Recommendations for further work to verify and to exploit the impulse response approach are made. AUTHOR

1.0 INTRODUCTION

The radar return from lunar and planetary surfaces is of interest for several reasons. For example, it can help in the determination of surface characteristics that may influence the establishment of design criteria for landing systems. This study was initiated to investigate the application of a new mathematical theory of the reflection of electromagnetic waves. The theory, which is described in Appendix I, is based on the derivation of the impulse response of a reflecting surface.[1] The approach is different from that of W. E. Brown.[2]

In this study, the theory is applied to a perturbed spherical surface to determine the probability distribution and spatial correlation of the slopes on the surface. Frequency-shift effects are not considered.

Because of the amount of experimental data available, the surface of the moon was chosen for this study. The technique might next be applied to Venus and to the Earth from about 250 kilometers.

This study suggests the computation of ensemble averages of return signals based upon assumptions of the statistics of the surface characteristics, rather than the computation of a single radar return signal for a particular orientation of the reflecting surface. In Sec. 2.0 of this report the Basore modified potential of a perturbed sphere in terms of surface parameters is derived. The modified potential is essentially the integral of the impulse response. The surface parameters used are the radius and angles determined by the deviation of the outer normal from that of a smooth sphere. Section 3.0 is devoted to statistical moments of the modified potential as a function of the statistics of the surface parameters. The moments of the modified potential are in turn used in Sec. 4.0 to describe ensemble averages of radar return signals.

---

[1] Basore, B. L., <u>The Impulse Response of a Reflecting Surface</u>, Report No. DTR-4, The Dikewood Corp., 5 January 1962.

[2] Brown, W. E., Jr., "A Lunar and Planetary Echo Theory". <u>Journal of Geophysical Research</u>, Vol. 65, pp 3087-3095.

The general approach to computing the mean envelope and the autocorrelation of the envelope of the return signal is given in Sec. 5.0. Listings of the two computer programs written are given in Appendix II. Finally, Sec. 6.0 shows some of the computed results for the moon. The results obtained are compared with experimental data. This Section also recommends some future work.

2.0 DEVELOPMENT OF THE MODIFIED POTENTIAL FOR A PERTURBED SPHERE

The impulse response function as derived in Appendix I is

$$h(t) = \frac{\lambda}{(4\pi)^2} \left\{ \frac{\partial}{\partial t} \left[\frac{1}{R^2} \int_C kG \cot(\vec{n}, \vec{R})\, ds \right]_{R = \frac{ct}{2}} \right\} \tag{1}$$

where

C is the integration path and is the set of points at distance R from the transmitter,
G is the antenna gain in the direction of the increment ds,
k is the voltage reflection coefficient of the surface at each point on the path,
$\vec{n}, \vec{R}$ is the angle between a vector from the transmitter to a point on the path and the normal to the surface at that point, and
λ is the wavelength of the transmitted signal.

The wavelength enters in the equation from the effective area of the matched transmitting-receiving antenna, $G\lambda^2/4\pi$. For computation, the portion inside the square brackets of Eq. (1) is considered first. It is denoted B(R) and is called the modified potential because of its relationship to the vector potential discussed in Appendix I. The following subsections develop a description of the different terms that make up B(R) for a rough sphere.

2.1 The Modified Potential

The first problem is to express B(R) in terms of parameters suitable for describing a rough surface. The geometry to be used is shown in Fig. 1. At a point on the contour, the normal can be described in terms of the direction cosines as illustrated in Fig. 2. The cosines of α_1, α_2, and α_3 are the projections of the normal on the on the r_o, θ_o, ϕ_o axes respectively. The normal $\vec{n}$ is then

$$\vec{n} = \cos\alpha_1 \vec{r}_o + \cos\alpha_2 \vec{\theta}_o + \cos\alpha_3 \vec{\phi}_o \tag{2}$$

To express $\vec{n}$ in parameters descriptive of the surface, two other angles γ and ξ are defined. The angle γ is the deviation from the r_o axis of the projection of $\vec{n}$ on the r_o, θ_o plane. The angle ξ is the deviation from the r_o axis of the projection of the normal on the r_o, θ_o plane. These angles may be described in terms of direction cosines as

$$\tan \gamma = \frac{\cos \alpha_2}{\cos \alpha_1} \tag{3a}$$

$$\tan \xi = \frac{\cos \alpha_3}{\cos \alpha_1} \tag{3b}$$

From Fig. 1, the R vector from the source to a point on the surface is in the r_o, θ_o plane and is described by

$$\vec{R} = R(\cos\delta\, \vec{r}_o + \sin\delta\, \vec{\theta}_o) \tag{4}$$

The cot $(\vec{n}, \vec{R})$ can be derived from the scalar product of $\vec{n}$ and $\vec{R}$ which gives

$$\cot(\vec{n}, \vec{R}) = \frac{\cos\delta\cos\alpha_1 + \sin\delta\cos\alpha_2}{\left\{1-(\cos\delta\cos\alpha_1 + \sin\delta\cos\alpha_2)^2\right\}^{\frac{1}{2}}} \tag{5}$$

Substitution of Eq. (3) into Eq. (5) yields

$$\cot(\vec{n}, \vec{R}) = \frac{\cos\delta + \sin\delta\tan\gamma}{(\sin\delta - \cos\delta\tan\gamma)^2 + \tan^2\xi} \tag{6}$$

It is desirable to convert the integration over s to an integration over the azimuthal angle ϕ. The usual relation for polar coordinates is

$$\left(\frac{ds}{d\phi}\right)^2 = \left(\frac{dr}{d\phi}\right)^2 + r^2\left(\frac{d\theta}{d\phi}\right)^2 + r^2\sin^2\theta \tag{7}$$

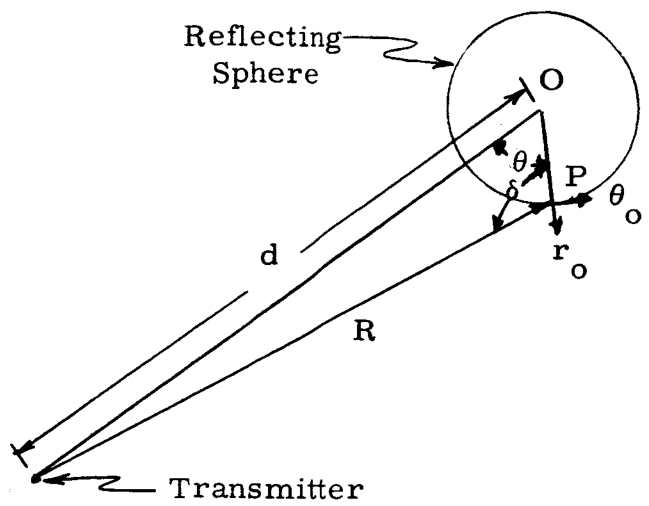

Fig. 1 - Geometry of Sphere and Source

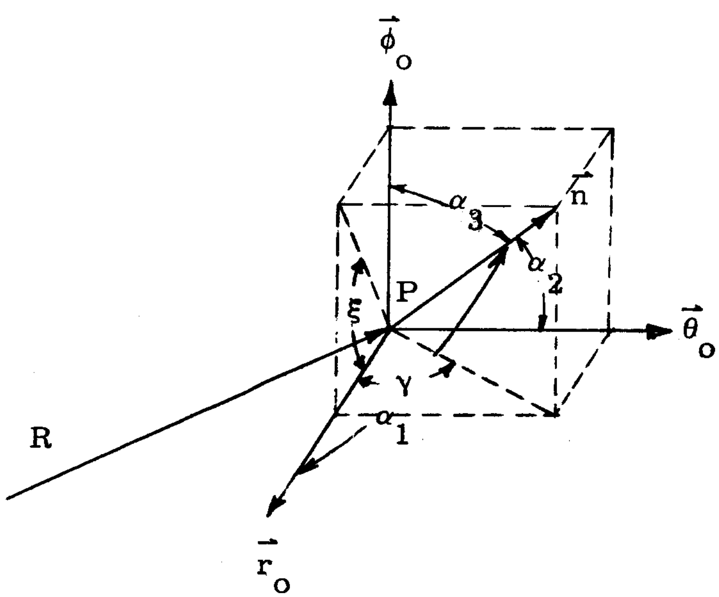

Fig. 2 - Relationship at a Point

To evaluate Eq. (7), consider the equation of the perturbed sphere centered on the origin as

$$G = r - r(\phi, \theta) = 0 \tag{8}$$

The relationships between the gradient of G and the normal yield the partial derivatives of r as

$$r_\theta = -r\frac{\cos \alpha_2}{\cos \alpha_1} = -r \tan \gamma \tag{9a}$$

$$r_\phi = -r \sin \theta \frac{\cos \alpha_3}{\cos \alpha_1} = -r \sin \theta \tan \xi \tag{9b}$$

The other relationship needed is that relating R to the intersection. This can be written as

$$F = r^2 + d^2 - 2rd \cos \theta - R^2 = 0 \tag{10}$$

Jacobians involving F and G give the result

$$\frac{dr}{d\phi} = \frac{r_\phi \, r \tan \delta}{r_\theta + r \tan \delta} \tag{11a}$$

$$\frac{d\theta}{d\phi} = - \frac{r_\phi}{r_\theta + r \tan \delta} \tag{11b}$$

Incorporation of Eqs. (9) and (11) into Eq. (7) yields the result

$$\left(\frac{ds}{d\phi}\right)^2 = r^2 \sin^2 \theta \; \frac{(\sin \delta - \tan \gamma \cos \delta)^2 + \tan^2 \xi}{(\sin \delta - \tan \gamma \cos \delta)^2} \tag{12}$$

Finally, the desired result is

$$\cot(\vec{n}, \vec{R}) \frac{ds}{d\phi} = r \sin \theta \frac{\cos \delta + \sin \delta \tan \gamma}{\sin \delta - \tan \gamma \cos \delta} \tag{13}$$

and the modified potential is

$$B(R) = \frac{1}{R^2} \int_0^{2\pi} kGr \sin \theta \frac{(\cos \delta + \sin \delta \tan \gamma)\, d\phi}{\sin \delta - \tan \gamma \cos \delta} \tag{14}$$

Note that a pole exists when $\tan \delta = \tan \gamma$, which occurs when $d\phi/ds = 0$. The only true pole for $B(R)$ in the γ, ξ plane occurs when $\tan \delta = \tan \gamma$ and $\tan \xi = 0$. However, this pole has a finite integral and is tractable mathematically. The method of integration used is described in Sec. 5.0.

2.2 Antenna Gain

The gain of the antenna is considered in the computer codes as a constant since the target sphere subtends a sufficiently small angle at the transmitter. In other cases the consideration of the variation of antenna gain can be easily included.

2.3 Reflection Coefficient

The k in the equations for the modified potential is a voltage reflection coefficient. For normal incidence on a smooth dielectric material, the reflection coefficient is simply

$$k = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \tag{15}$$

where η represents the usual impedance. However, for the problem considered here oblique incidence occurs and both the angle of incidence and the polarization angle must be considered. The reflection coefficient for the parallel and the perpendicular components will be termed k_H and k_V respectively. Employment of the usual conditions for the reflection of an EM wave at the boundary of a smooth homogeneous dielectric yields

$$k_H = k_1 \cos^2(\phi - \xi) + k_2 \sin^2(\phi - \xi) \tag{16a}$$

$$k_V = (k_1 - k_2) \sin(\phi - \xi) \cos(\phi - \xi) \tag{16b}$$

where $\phi = 0$ has been arbitrarily selected as the azimuthal angle in the direction of the polarization. The k's in Eq. (16) are

$$k_1 = \frac{\cos(\vec{n}, \vec{R}) - \rho_1 \sqrt{1 - \rho_2^{\,2} \sin^2(\vec{n}, \vec{R})}}{\cos(\vec{n}, \vec{R}) + \rho_1 \sqrt{1 - \rho_2^{\,2} \sin^2(\vec{n}, \vec{R})}} \tag{17a}$$

$$k_2 = \frac{\sqrt{1 - \rho_2^{\,2} \sin^2(\vec{n}, \vec{R})} - \rho_1 \cos(\vec{n}, \vec{R})}{\sqrt{1 - \rho_2^{\,2} \sin^2(\vec{n}, \vec{R})} + \rho_1 \cos(\vec{n}, \vec{R})} \tag{17b}$$

where

$$\rho_1 = \sqrt{\frac{\mu_2 \epsilon_1}{\epsilon_2 \mu_1}} \quad \text{and} \quad \rho_2 = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$$

The validity of the equations for reflection coefficient is certainly open to question, since they are derived for a smooth homogeneous dielectric, but the effect of possible error here is not thought to be serious.

3.0 STATISTICAL RELATIONSHIPS

The equation given for B(R) in the preceeding Section is, of course, valid only for a specified path. That is, γ, ξ, and r must be specified for every point on the path to obtain a value for B(R). However, some characteristics of B(R) can be described statistically. The statistical relationships necessary for the description of averages of the return signal are developed below.

3.1 Moments of B(R)

Of major interest is the ensemble average of B(R) and the autocorrelation $\overline{B(R)B(R+\Delta R)}$. These functions are, in part, specified by the surface parameters. Since the relative deviation of the radius r from the mean radius a is small, the mean radius will be used instead of r. This leaves only γ and ξ as variables.

The simplest way to view the ensemble average is to consider the integral of Eq. (1) as the limit of the sum

$$B(R) = \frac{1}{R^2} \lim_{n\to\infty} \sum_{i=1}^{n} [k_i G_i \cot(\vec{n}_i, \vec{R})] \Delta s_i \tag{18}$$

Since expectation is a linear operator, and G is independent of the surface parameters,

$$\overline{B(R)} = \frac{1}{R^2} \lim_{n\to\infty} \sum_{i} G_i \overline{k_i \cot(\vec{n}_i, \vec{R})} \Delta s_i \tag{19}$$

The variables γ and ξ are assumed independent of each other at a point; thus,

$$\overline{B(R)} = \frac{1}{R^2} \int_C G \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k \cot(\vec{n}, \vec{R})\, p(\gamma)\, p(\xi)\, d\gamma d\xi ds \tag{20}$$

obtains. The autocorrelation of B(R) can be obtained in a similar fashion and

$$\overline{B(R)B(R+\Delta R)} = \frac{1}{R^2(R+\Delta R)^2} \int_{C_1} \int_{C_2} G_1 G_2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k_1 \cot(\vec{n}_1, \vec{R})$$

$$k_2 \cot(\vec{n}_2, \overrightarrow{R+\Delta R})\, p(\gamma_1, \gamma_2, \xi_1, \xi_2)\, d\gamma_1\, d\gamma_2\, d\xi_1\, d\xi_2\, ds_1\, ds_2 \qquad (21)$$

The joint probability density function in Eq. (21) is a function of the particular values of s_1 and s_2 as well as of the range difference ΔR. The matrix of correlation coefficients is derived later in this section. In the above equations, γ and ξ were treated as the statistical variables. In the actual computations, the tangents of the angles are employed. The tangents are used since they occur naturally in the equation for B(R) and have an infinite range.

3.2 Statistical Variables

The variables of interest are the tangents of the deviation angles γ and ξ. The deviation of the radius of the perturbed sphere from the mean will not be included initially. The probability density functions of the tangents of the variables will be approximated by the weighted sum of a series of Gaussian functions each having a zero mean, i.e., of the form

$$p(x) = \sum_i W_i N(0, \sigma_i; x); \sum_i W_i = 1 \qquad (22)$$

This series was selected since it is symmetric, has an infinite range (as does the tangent), has a mode of zero, and is a member of the class of density functions that requires only linear correlation coefficients to describe a multivariate distribution.

The correlation coefficients are a function of the displacement (along a great-circle path) and the angular rotation on ϕ. Consider two points on an integration path. The deviation angles at these points are γ_1, ξ_1, and γ_2, ξ_2. Through these two points a great circle is passed. Two new deviation angles a and b are defined where a is the deviation in the plane of the great circle and b is the deviation in a plane orthogonal to the great circle. The angle between the slope of a_1 and ξ_1 is $\phi/2$. The relationship between the slopes may be written

$$\begin{pmatrix} \tan a_1 \\ \tan b_1 \end{pmatrix} = \begin{pmatrix} \cos \phi/2 & -\sin \phi/2 \\ \sin \phi/2 & \cos \phi/2 \end{pmatrix} \begin{pmatrix} \tan \xi_1 \\ \tan \gamma_1 \end{pmatrix}$$

and

$$\begin{pmatrix} \tan \xi_2 \\ \tan \gamma_2 \end{pmatrix} = \begin{pmatrix} \cos \phi/2 & -\sin \phi/2 \\ \sin \phi/2 & \cos \phi/2 \end{pmatrix} \begin{pmatrix} \tan a_2 \\ \tan b_2 \end{pmatrix}$$

It is assumed that the correlation between $\tan a_i$ and $\tan b_j$ is zero for any ij, that $\overline{\tan a_1 \tan a_2}$ is equal to $\overline{\tan b_1 \tan b_2}$, and that the correlation is only a function of distance along the great-circle path. Using the above relationships and assumptions, the following matrix of correlation coefficients was obtained.

	ξ_1	ξ_2	γ_1	γ_2
ξ_1	1	$\rho \cos \Delta\phi$	0	$\rho \sin (\Delta\phi)$
ξ_2	$\rho \cos (\Delta\phi)$	1	$-\rho \sin (\Delta\phi)$	0
γ_1	0	$-\rho \sin (\Delta\phi)$	1	$\rho \cos (\Delta\phi)$
γ_2	$\rho \sin (\Delta\phi)$	0	$\rho \cos (\Delta\phi)$	1

The coefficient ρ is more properly $\rho(D)$ where D is the displacement and is of the form

$$\rho = \frac{\overline{\tan a_1 \tan a_2}}{\sigma^2 (\tan a)}$$

If the distribution of tan a (or the tangent of γ or ξ) and the correlation coefficient is known, the multivariate equivalent of Eq. (22) can be written. Letting x indicate the array ($\tan \xi_1$, $\tan \xi_2$, $\tan \gamma_1$, $\tan \gamma_2$), the multivariate distribution is

$$P(x) = \sum_i \frac{W_i}{(2\pi)^2 \sigma_i^4 |M|^{\frac{1}{2}}} \exp \left[-\frac{1}{2\sigma_i^2} (\tilde{x} M^{-1} x) \right] \tag{23}$$

where M represents the covariance matrix.

3.3 A Relationship of the Radius to the Slope

The approach described in this report is directed toward determination of the probability density and spatial correlation of the slope. However, it is possible to describe partially the surface elevation or radius from these quantities. If the mean radius is a and the deviation from the mean is x, then $r = a + x$ and x has a mean of zero. In general, for two points separated by a distance D (along a great-circle path),

$$\overline{(x_2 - x_1)^2} = 2\sigma_x^2 [1 - \rho_x (D)] \tag{24}$$

If ψ is the deviation angle of the normal in the great-circle plane, then

$$x_2 - x_1 = \int_{P_1}^{P_2} \tan \psi \, ds \tag{25}$$

The path of integration is the intersection of the great-circle plane and the sphere. The variance of the height differential can be written

$$\overline{(x_2 - x_1)^2} = \int_0^D \int_0^D \overline{\tan\psi_1 \tan\psi_2} \; ds_1 \, ds_2$$

$$= \sigma_\psi^2 \int_0^D \int_0^D \rho_\psi (s_2 - s_1) \, ds_1 \, ds_2 \tag{26}$$

for ρ_ψ as a function of the displacement between two points. The transformation $\Delta s = s_2 - s_1$ gives

$$\overline{(x_2 - x_1)^2} = 2\sigma_\psi^2 \int_0^D \int_0^s \rho_\psi (\Delta s) \, d\Delta s \, ds \tag{27}$$

Combining Eqs. (24) and (27) gives the relationship

$$\sigma_x^2 \left[1 - \rho_x(D) \right] = \sigma_\psi^2 \int_0^D \int_0^s \rho_\psi (\Delta s) \, d\Delta s \, ds \tag{28}$$

Thus, if the standard deviation and correlation coefficient of the slope are known, the same parameters can be determined for the radius. In the limit, Eq.(28) is a form of Daniels' relationship.[3]

---

[3] Daniels, F. B., "A Theory of Radar Reflection from the Moon and Planets", Journal of Geophysical Research, Vol. 66, No. 6, p 1784.

$$\sigma_{\psi}^{2} = \lim_{D \to 0} \frac{2\sigma_x^2}{D^2} \left[1 - \rho_x (D) \right] \qquad (29)$$

If the angular correlation coefficient can be determined, the variations in height can be described partially.

4.0 THE RETURN SIGNAL

The relationships necessary to describe a radar return signal were developed in the previous section. These relationships and the general characteristics of an impulse response are employed below to develop several functions of the return signal.

4.1 Description of the Return Signal

Given an impulse response h(t), the return signal g(t) can be expressed as the convolution of h(t) with the transmitted signal f(t), as follows

$$g(t) = \int_{t_1}^{t_2} f(t - \tau)\, h(\tau)\, d\tau \qquad (30)$$

if a pulse is transmitted and its duration T is less than a/c where a is the radius of the planet and c is the velocity of light, the limits of the integral are, defining t = 0 as the leading edge of the pulse:

t_1	t_2	
t_0	t	for $t_o \leq t < t_o + T$
$t - T$	t	for $t_o + T \leq t < t_m$
$t - T$	t_m	for $t_m \leq t < t_m + T$

where t_o is the transport time $2(d - a)/c$ to and from the nearest point on the reflecting surface, and t_m is the maximum time $2\sqrt{d^2 - a^2}\,/\,c$. Integration of Eq. (30) by parts gives the result

$$g(t) = -\frac{\lambda}{(4\pi)^2} \int_{t_1}^{t_2} \frac{df(t - \tau)}{d\tau}\, B(\tau)\, d\tau \qquad (31)$$

$B(\tau)$ may be expressed as a Taylor series expansion about $\tau = t$, in the interval from t_1 to t_2, as follows:

$$B(\tau) = \sum_{n=0}^{\infty} \frac{1}{n!} \left.\frac{d^n B(\tau)}{d\tau^n}\right|_{\tau = t} (\tau - t)^n$$

Then

$$g(t) = -\frac{\lambda}{(4\pi)^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left.\frac{d^n B(\tau)}{d\tau^n}\right|_{\tau = t} \int_{t_1}^{t_2} \frac{df(t-\tau)}{d\tau} (\tau - t)^n \, d\tau$$

The first term, $n = 0$, of this series is

$$g_o(t) = -\frac{\lambda}{(4\pi)^2} B(t) \left[f(t - t_2) - f(t - t_1) \right]$$

Trial of the various values of t_1 and t_2 shows that this term is always zero. Using this fact and integrating by parts again in the reverse order, one obtains

$$g(t) = \frac{\lambda}{(4\pi)^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \left.\frac{d^n B(\tau)}{d\tau^n}\right|_{\tau = t} \int_{t-t_1}^{t-t_2} f(x)\, x^{n-1} \, dx \qquad (32)$$

where $x = t - \tau$. Furthermore, if it assumed that the transmitting antenna gain, as a function of frequency, is zero at zero frequency, then $\int_0^t f(x)\,dx = 0$, and for the middle time interval, $t_o + T < t < t_m$, the $n = 1$ term vanishes. Also, in the middle time interval, the integral in the n^{th} term is independent of t and can be expressed as $F_n(T)$, or

$$g_n(t) = (-1)^{n-1} F_n(T)$$

where

$$F_n(T) = \int_T^0 f(x)\, x^{n-1}\, dx$$

for the middle time interval. This implies that all time dependence of g(t) is due to the derivatives of $B(\tau)$ evaluated at $\tau = t$, and for $a \ll \lambda$, most of the energy in g(t) will be at frequencies much lower than ω, provided there are no rapid fluctuations in B within a pulsewidth.

Another interesting expression for g(t) is obtained by considering f(t) to be the product of an envelope function and a sinusoidal RF wave:

$$f(t) = \begin{cases} A(t) \sin \omega t & 0<t<T \\ 0 \text{ elsewhere} & \end{cases}$$

then

$$f(t - \tau) = \begin{cases} A(t - \tau) \sin \omega (t - \tau) & t-T<\tau< t \\ 0 \text{ elsewhere} & \end{cases}$$

and

$$\frac{df(t - \tau)}{d\tau} = \begin{cases} \omega A(t - \tau) \cos \omega (t - \tau) + \dfrac{dA(t - \tau)}{d\tau} \sin \omega (t - \tau) \\ \text{for } t - T < \tau < t, \text{ 0 elsewhere} \end{cases}$$

This permits Eq. (31) to be written as

$$g(t) = \frac{\lambda}{(4\pi)^2}\left\{\cos\omega\tau\int_{t_1}^{t_2}\left[\omega A(t-\tau)\cos\omega\tau - \frac{dA(t-\tau)}{d\tau}\sin\omega\tau\right]B(\tau)\,d\tau\right.$$

$$\left. + \sin\omega\tau\int_{t_1}^{t_2}\left[\omega A(t-\tau)\sin\omega\tau + \frac{dA(t-\tau)}{d\tau}\cos\omega\tau\right]B(\tau)\,d\tau\right\} \quad (33)$$

The four integrals involved may be identified as the integrals for evaluating the Fourier coefficients of the functions $\omega A(t-\tau)B(\tau)$ and $\frac{dA(t-\tau)}{d\tau}B(\tau)$, at the frequency ω. If A changes only slightly in a single cycle, then the coefficients will be small unless there are fluctuations in $B(\tau)$ in distances of the order of a wavelength. The power-series expansion of $B(\tau)$ may be inserted in Eq.(33) if desired.

The details of the computation of $g(t)$ are obviously quite dependent on the shape of the envelope function, $A(t)$. Consider the rectangular pulse with an integral number of cycles in it, so that

$A(t) = A$ and $\int_0^T f(t) = 0$. Then, from Eq. (31)

$$g(t) = \frac{cA}{8\pi}\int_{t_1}^{t_2}\cos\omega(t-\tau)\,B(\tau)\,d\tau$$

Using Eq. (32), the leading term for the first time interval is

$$g_1(t) = \frac{cA}{16\pi\omega^2}\left.\frac{dB(\tau)}{d\tau}\right|_{t-t}\cos\omega(t-t_o), \quad t_o \leq t < t_o + T \quad (34)$$

and a similar expression for the last time interval. However, for the middle interval $g_1(t)$ vanishes, and the leading term is

$$g_2(t) = \frac{c\,T\,A}{16\,\pi\,\omega^2}\,\frac{d^2B(\tau)}{d\tau^2}\Bigg|_{\tau - t} \tag{35}$$

This is not only a higher-order term, but it has a weak time dependence if $B(\tau)$ does not fluctuate. If it had not been assumed that there were an integral number of cycles in the pulse, there would have been a term due to $\frac{dB}{d\tau}$, but it would still have had the "weak" time dependence. Finally, application of Eq. (33) yields

$$g(t) = \frac{cA}{8\pi}\left\{\cos\omega t\int_{t_1}^{t_2}\cos\omega\tau\,B(\tau)\,d\tau + \sin\omega\tau\int_{t_1}^{t_2}\sin\omega\tau B(t)\,d\tau\right\} \tag{36}$$

and the Fourier coefficients of $B(\tau)$ are the significant quantities. It might seem that this expression would give terms at ω even for the middle interval, but because the limits of the integrals involve t if the power series for $B(t)$ is used, the time dependence of the resultant expression is still "weak".

4.2 Measures of the Return Signal

The return signal from a planet will usually vary widely from pulse to pulse. Consequently, it is often characterized by averages such as the mean power, the mean envelope, or the autocorrelation function. The most obvious average $\overline{g(t)}$ is easily obtained analytically by replacing $B(\tau)$ in the equations of the previous section by $\overline{B(\tau)}$. However, this measure contains both the RF modulated wave and any low frequency components of the return signal. An attempt has been made in the previous section to show that such components can exist, and perhaps even dominate the R F component in certain cases, especially for the middle time interval. A more interesting measure of $\overline{g(t)}$ is the average of the ω component, which will be called $\overline{g_\omega(t)}$. It has been shown, at least for the rectangular pulse, that $g_\omega(t)$ arises from the ω component of $B(t)$, $B_\omega(t)$, evaluated between the limits t_1 and t_2. An envelope detector attached to a resonant receiver actually measures the average over an RF cycle of $g_\omega(t)$, or $\left|\widetilde{g_\omega(t)}\right|$ where the wiggly bar represents RF averaging as distinguished from the straight bar for ensemble averages. That is, if a member of the ensemble of $g_\omega(t)$ is $g_{\omega i}(t) = E_i(t) \cos(\omega\tau + \psi i)$, the RF average is $\left|\widetilde{g_\omega i(t)}\right| = 2E_i(t)/\pi$. If the peak amplitude of the transmitted pulse is A, the normalized average envelope response is $\overline{E_i(t)/A}$, which will be called $E(t)$.

All the processes involved in obtaining $E(t)$ from $g(t)$ are linear operations except the taking of the absolute value. It would be desirable to be able to express $E(t)$ in terms of $\overline{B(\tau)}$ (since $\overline{B(\tau)}$ is in turn expressed in terms of the probability distribution of the deviation angles without involving the autocorrelation function of the deviation angles). If $E(t)$ can be adequately expressed in terms of $\overline{B(\tau)}$, then comparison of $E(t)$ with experimental values can determine the W_i's and σ_i's of Eq. (22). This is not possible in general because of the absolute-value operation involved. The best that can be done is to separate $E(t)$ into two parts, one due solely to $\overline{B(\tau)}$, and called $E_c(t)$, and another called $E_r(t)$ that vanishes when the random component of $B(\tau)$ vanishes. $E_o(t)$ will be investigated to see whether there are important cases where it is the dominant part of the envelope function. This will be accomplished by studying the $\overline{g(t)}$ generated by $\overline{B(\tau)}$.

$\overline{B(\tau)}$ and its derivatives are smoothly varying functions between t_o and t_m. It and its second derivative are positive, and its first derivative is negative. If $\lambda \ll cT \ll a$ then $\overline{B(\tau)}$ changes very little in one RF cycle. This makes expressions such as Eq. (7) very difficult to evaluate numerically. On the other hand, the relatively slow variation of $\overline{B(\tau)}$ with τ makes the Taylor series expansion converge rapidly. In fact, for the numerical examples corresponding to lunar experiments, to be discussed in a later section, the series was well represented by its leading term. Therefore Eqs. (34) and (35) were used to evaluate $\overline{g(t)}$ in the outside and middle time intervals, respectively. For the outside time intervals the equation is already in the form necessary for evaluating $E_o(t)$, since it is the product of a slowly varying time function and a $\cos \omega(t - t_o)$ factor.

Thus

$$E_o(t) = \frac{c}{16 \pi \omega^2} \left| \overline{\frac{dB(\tau)}{d\tau}} \right| \Bigg|_{\tau = t} \qquad (37)$$

for the outside time intervals. For the inside time interval, the only component received will be the Fourier component of the second derivative of $\overline{B(\tau)}$ at ω or within the bandwidth of the receiver. Numerical analysis shows that this is not the major component of the experimentally observed signal. Therefore, in this time interval, it is not justified to neglect $E_r(t)$. In the first time interval the numerical value of $E_o(t)$ is the same order of magnitude as the observed signal, so Eq. (37) may be useful, at least for some lunar problems.

There is no practical analytical method known to the authors for evaluating the random component of $B(\tau)$ directly, and hence obtaining $E_r(t)$. For cases where this is important, it will be necessary to use some other measure of the return signal for comparison with experiment. The next possibility to be considered is $g_\omega^2(t)$. It has the advantage of not requiring the absolute magnitude signs that occur in defining the envelope function. Its disadvantage is that an assumption of the correlation function of the deviation angles is required. For the unit amplitude rectangular pulse, one obtains

$$\overline{g^2(t)} = \left(\frac{c}{8\pi}\right)^2 \left[\overline{\int_{t_1}^{t_2} \cos \omega(t - \tau) \, B(\tau) \, d\tau} \right]^2$$

which can be expressed as

$$g^2(t) = \left(\frac{c}{8\pi}\right)^2 \int_{t_1}^{t_2} \int_{t_1-\tau}^{t_2-\tau} \cos\omega(t-\tau)\cos\omega(t-\tau-u)$$

$$\overline{B(\tau+u)B(\tau)}\, du\, d\tau \qquad (38)$$

The limits t_1 and t_2 are defined as before. In this case the ω component at the antenna appears as a 2ω component after squaring. By using a bivariate Taylor expansion of $\overline{B(\tau+u)B(\tau)}$ and selecting the 2ω component, one obtains $\overline{g_\omega^2}$. The leading terms of the expansion are

$$\overline{B(\tau+u)B(\tau)} = \overline{B^2}(t) + u\left.\frac{d[\overline{B(\tau+u)B(\tau)}]}{du}\right|_{\substack{u=0\\ \tau=t}}$$

$$+(\tau-t)\left.\frac{d[\overline{B(\tau+u)B(\tau)}]}{d\tau}\right|_{\substack{u=0\\ \tau=t}} \qquad (39)$$

The $\overline{B^2}(t)$ term vanishes when $\overline{g^2}(t)$ is computed, and the term due to the derivative with respect to τ behaves in essentially the same way as the derivative of $\overline{B(\tau)}$ term did in computing $\overline{g}(t)$, vanishing in the middle time interval. The term due to the derivative with respect to u is the term that corresponds to the random component of g (t). If this derivative with respect to u is large enough to cause an appreciable change in $\overline{B(\tau+u)B(\tau)}$ for u's corresponding to times on the order of one RF cycle, then this term will cause the major component of $\overline{g_\omega^2}(t)$, at least for the middle time interval.

The last measure of g(t) to be considered is the autocorrelation of g (t), $\overline{g(t)\, g(t+\sigma)}$. This functions allows the consideration of the autocorrelation of $B(\tau)$ over ranges greater than T. However, the discussion of $\overline{g^2(t)}$ indicates that the important autocorrelations of $B(\tau)$ for creating components of $g(t)$ at ω are those with separation distances of at most a few R F cycles. This time interval is too short to be resolvable in the experimental data, so little new information is to be expected from $\overline{g(t)\, g(t+\sigma)}$. For these reasons the discussion will be confined to setting down the defining relations. Because calculation of $\overline{g^2(t)}$ involves doing most of the necessary work for calculating $\overline{g(t)\, g(t+\sigma)}$, the machine code was designed to calculate the more general measure. The autocorrelation function may be written as

$$\overline{g(t)\, g(t+\sigma)} = \left(\frac{c}{8\pi}\right)^2 \int_{t_1}^{t_2} \int_{t'_1}^{t'_2} \cos\omega(t-v)\cos\omega(t+\sigma-u)$$

$$\overline{B(v)\, B(\mu)}\, d\,u\, d\,v \tag{40}$$

The major difference from the expression for $\overline{g^2(t)}$ is in the limits integration. These are now given by the table on the following page.

t range		V limits		M limits	
		$0 < \sigma < T$			
t_o	$t_o + T - \sigma$	t_o	t	t_o	$t + \sigma$
$t_o + T - \sigma$	$t_o + T$	t_o	t	$t + \sigma - T$	$t + \sigma$
$t_o + T$	$t_m - \sigma$	$t - T$	t	$t + \sigma - T$	$t + \sigma$
$t_m - \sigma$	t_m	$t - T$	t	$t + \sigma - T$	t_m
t_m	$t_m + T - \sigma$	$t - T$	t_m	$t + \sigma - T$	t_m
		$T < \sigma < t_m - t_o - T$			
t_o	$t_o + T$	t_o	t	$t + \sigma - T$	$t + \sigma$
$t_o + T$	$t_m - \sigma$	$t - T$	t	$t - T + \sigma$	$t + \sigma$
$t_m - \sigma$	$t_m + T - \sigma$	$t - T$	t	$t + \sigma - T$	t_m
		$t_m - t_o - T < \sigma < t_m - t_o$			
t_o	$t_m - \sigma$	t_o	t	$t + \sigma - T$	$t + \sigma$
$t_m - \sigma$	$t_o + T$	t_o	t	$t + \sigma - T$	t_m
$t_o + T$	$t_m + T - \sigma$	$t - T$	t	$t + \sigma - T$	t_m
		$t_m - t_o < \sigma < t_m + T - t_o$			
t_o	$t_m + T - \sigma$	t_o	t	$t + \sigma - T$	t_m

5.0 COMPUTING APPROACH

The theory of the preceeding sections has been used in the preparation of two computer programs or codes. The first code was designed to investigate the envelope function from $\overline{B(\tau)}$. The second calculates $\overline{g^2(t)}$ and $\overline{g(t)\,g(t+\sigma)}$ from $\overline{B(\tau)\,B(\tau+u)}$. The first code has been checked out and compared briefly with experiment, but time did not permit this for the second code. Listings of the two codes appear in Appendix II.

5.1 The Envelope Code

A Monte Carlo approach was selected to evaluate the integral expressions for the measures of $B(R)$ (in the case of this code $\overline{B(R)}$ as expressed by Eq. (20)). This selection was made since the problem is essentially a statistical one. In addition, there are three other reasons for this choice. The first is that n-dimensional integrals can often be evaluated in fewer steps if points are selected for evaluation of the integrand randomly throughout the n-dimensional space, and the integral is evaluated as the average value of the integrand times the area of the surface or hypersurface of integration.

The other two reasons for choosing the Monte Carlo approach relate to the singularity of the integrand. When $B(R)$ was expressed in the form of an integral over ϕ between 0 and 2π in Eq. (14) a singularity occurred whenever $\tan\delta = \tan\gamma$. Tests showed that in this form the integral diverges. For this reason, the integrals in Sec. 3 were used in the form of integrals over s. In this form there is still a singularity when the normal $\vec{n}$ is directed along $\vec{R}$, but this only occurs for the double condition, $\tan\delta = \tan\gamma$ and $\tan\xi = 0$, and the integral converges. However, in this form the limits of integration are not known, since the length of the contour is not known in advance. It was found that by taking a random walk around the contour, the contour length could be evaluated at the same time contributions from various parts of the contour to the integral were being evaluated.

The last reason for using a Monte Carlo method is that the integrand still had a singularity even in the form in which it converged. To accurately and efficiently evaluate an integral with a singular integrand, many more mesh points or samples are needed in the vicinity of the singularity. In the Monte Carlo method this can be accomplished by biasing the probability distribution from which the variables are picked. The contribution to the integrand by this factor is then removed before accumulation. This corresponds to a nonuniform and variable mesh spacing in ordinary methods of numerical integration which is difficult to formulate for multiple integration.

Equation (20) may be rewritten as

$$\frac{R^2}{G} \frac{d[\overline{B(R)}]}{ds} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} k \cot(\vec{n}, \vec{R}) P(\tan\gamma) P(\tan\xi)\, d(\tan\gamma) d(\tan\xi)$$

To execute the biasing operation, the variables of integration are changed to

$$dQ_\gamma = \frac{P(\tan\gamma)}{\sqrt{\tan\gamma - \tan\delta}} \, d(\tan\gamma)$$

$$dQ_\xi = \frac{P(\tan\xi)}{\sqrt{\tan\xi}} \, d(\tan\xi)$$

The Q's are actually biased cumulative distributions. The lower limits of integration become zero, and the upper limits become

$$D_\gamma = \int_{-\infty}^{\infty} \frac{P(\tan\gamma)}{\sqrt{\tan\gamma - \tan\delta}} \, d(\tan\gamma)$$

$$D_\xi = \int_{-\infty}^{\infty} \frac{P(\tan\xi)}{\sqrt{\tan\xi}} \, d(\tan\xi)$$

The reciprocal square root of the distance from the singularity is chosen as the bias factor to approximately compensate for the strength of the singularity. Thus one obtains

$$\frac{R^2}{G}\ \frac{d[\overline{B(R)}]}{ds} = \int_0^{D_\gamma}\int_0^{D_\xi} k \cot(\vec{n}, \vec{R})\sqrt{\tan\gamma - \tan\delta}\,\sqrt{\tan\xi}\ dQ_\gamma\, dQ_\xi$$

If Q_γ and Q_ξ are chosen independently from uniform distributions N times, and the integral is approximated by the average value of the integrand times the product of the ranges, one obtains

$$\frac{R^2}{G}\ \frac{d\,[\overline{B(R)}]}{ds} = \frac{D_\gamma D_\xi}{N}\sum_{i=1}^{N} k_i \cot(\vec{n}_i \vec{R})\sqrt{\tan\gamma_i - \tan\delta}\ \sqrt{\tan\xi_i}$$

where N is still arbitrary. If a Δs is selected and for each selection of Q_γ and Q_ξ, a corresponding $\Delta\phi$ is calculated assuming $\Delta\phi = \frac{d\phi}{ds}\Delta s$, then if ϕ is started at 0 and accumulated, the series of selections may be terminated at $\phi = 2\pi$. The estimate of the length of the contour is then $\overline{S(R)} = N\Delta s$, and

$$\overline{B(R)} = N\Delta s\ \frac{d[\overline{B(R)}]}{ds}$$

$$= \frac{\Delta s\, D_\gamma D_\xi G}{R^2}\sum_{i=1}^{N} k_i \cot(\vec{n}_i\vec{R})\sqrt{\tan\gamma_i - \tan\delta}\sqrt{\tan\xi_i} \qquad (41)$$

If this process is repeated N_c times to improve the estimate of $\overline{B(R)}$, then the best estimate of $\overline{B(R)}$ is the average of the individual estimates. If the desired estimate of $\overline{B(R)}$ is to be the average value over a range of R such as the pulsewidth, then R may be chosen from a uniform distribution on its range for each of the N_c estimates. Thus

$$\overline{B(R)} = \frac{G D_\xi}{N_c} \sum_{j=1}^{N_c} \frac{D_{\gamma j} \Delta s_j}{R_j^2} \sum_{i=1}^{N} k_{ij} \cot(\vec{n}_{ij}, \vec{R}_j) \sqrt{\tan \gamma_{ij} - \tan \delta_j} \sqrt{\tan \xi_{ij}}$$

$$= \frac{G D_\xi}{N_c} \sum_{i=1}^{NN_c} \frac{D_{\gamma i} \Delta s_i k_i}{R_i^2} \cot(\vec{n}_i, \vec{R}_i) \sqrt{\tan \gamma_i - \tan \delta_i} \sqrt{\tan \xi_i} \quad (42)$$

D_γ and Δs have been included in the summation since D_γ is dependent on $\tan \delta$ which is in turn dependent on R, and the most efficient choice of Δs is, in general, dependent on $\tan \delta$. If Δs is to be chosen so that there are at least N_{min} steps around the contour, then

$$\Delta s = \frac{2\pi a \sin\theta}{N_{min}}$$

However, it was found that when the maximum σ of the probability distribution was larger than $\tan \delta$, an inefficiently large number of steps were required to get around a contour. Therefore, for $2\sqrt{2}\,\sigma_{max} > \tan \delta$, Δs was replaced by $2\sqrt{2}\sigma_{max} \Delta s / \tan \delta$.

The variance of the estimate of $\overline{B(R)}$ is given by

$$\sigma^2 (\overline{B(R)}) = \frac{1}{N_c(N_c - 1)} \sum_{j=1}^{N_c} \left(\overline{(B(R)}_{ij} \right)^2 - N_c \left(\overline{B(R)} \right)^2 \quad (43)$$

where $(\overline{B(R)}_j)^2$ is the square of Eq. (41), and $(\overline{B(R)})^2$ is the square of Eq. (42). Each of the above equations is actually to be considered as two equations, one for evaluating the depolarized component of B(R), $B_V(R)$, using k_V for k from Eq. (16b), and the other for determining the undepolarized component of B(R), $B_H(R)$, using Eq. (16a). The evaluation of $\cot(\vec{n}, \vec{R})$, and of $\frac{d\phi}{ds}$ (for determining $\Delta\phi$) proceed from Eqs. (13) and (12) respectively.

The plan of the calculations to this point is then to evaluate Q versus $\tan\xi$ from Eq. (22) with W_i and σ_i as input data. The range R is then chosen, and the trigometric functions of δ and θ are computed starting from the law of cosines. The angle ϕ is set equal to 0, Δs is evaluated, and Q_γ is evaluated versus $\tan\delta$. Then $\tan\gamma$ and $\tan\xi$ are chosen by choosing Q_γ and Q_ξ from a uniform distribution. This allows evaluation of one term of Eq. (41), and the evaluation of a $\Delta\phi$. ϕ is incremented and a new selection of $\tan\gamma$ and $\tan\xi$ is made. This process is continued until ϕ reaches 2π, accumulating the contributions to $\overline{B(R)}$. A term of Eq. (43) is calculated and a new R is calculated; this process is iterated N_c times, continuing the accumulation of $\overline{B(R)}$, and $\sigma^2\overline{(B(R))}$.

After evaluation of $\overline{B(R)}$ and $\sigma^2\overline{(B(R))}$ for each of several pulse positions, $g_o(t)$ and its variance are calculated using Eqs. (35) and (36). The derivatives are estimated from the finite difference approximation over the interval between the pulse positions. It is realized that $g_o(t)$ is not necessarily a good approximation to the total signal received at frequency ω at all times. The main purpose of the calculation is to see whether useful comparisons with experiment can be made without including the random component of B(R).

An additional output is available from this code, namely the estimate of the contour length $\overline{S(R)}$ at each R used. As will be explained below, having this contour length simplifies the evaluation of $\overline{B(R)\,B(R+\Delta R)}$ in the other code.

5.2 The Correlation Code

The calculation of $\overline{B(R)B(R+\Delta R)}$ proceeds from Eq. (21)

rewritten as

$$\overline{B(R)B(R+\Delta R)} = \frac{1}{R^2(R+\Delta R)^2} \int_{\phi_1=0}^{\phi_1=2\pi} \int_{\phi_2=0}^{\phi_2=2\pi} G_1 G_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$

$$k_1 k_2 \cot(\vec{n}_1, \vec{R}) \cot(\vec{n}_2, \overrightarrow{R+\Delta R})\, P(\tan\gamma_1, \tan\gamma_2, \tan\xi_1, \tan\xi_2) d(\tan\gamma_1)$$

$$d(\tan\gamma_2)\, d(\tan\xi_1)\, d(\tan\xi_2)\, ds_1\, ds_2$$

The transformation of variables used is analogous to that for $\overline{B(R)}$, and is given by

$$dQ_{\gamma_1} = \frac{P(\tan\gamma_1)}{\sqrt{\tan\gamma_1 - \tan\delta_1}} \quad d(\tan\gamma_1)$$

$$dQ_{\gamma_2} = \frac{P(\tan\gamma_2)}{\sqrt{\tan\gamma_2 - \tan\delta_2}} \quad d(\tan\gamma_2)$$

$$dQ_{\xi_1} = \frac{P(\tan\xi_1)}{\sqrt{\tan\xi_1}} \quad d(\tan\xi_1)$$

$$dQ_{\xi_2} = \frac{P(\tan\xi_2)}{\sqrt{\tan\xi_2}} \quad d(\tan\xi_2)$$

The upper limits, $D_{\gamma 1}$, $D_{\gamma 2}$, $D_{\xi 1}$, and $D_{\xi 2}$ are again the integral of the corresponding d Q's from $-\infty$ to $+\infty$. These transformations are specified on the basis of the univariate distributions because it is difficult to choose variables from a multivariate distribution on the basis of a group of random numbers. If the length of the contours has been determined using the envelope code, the limits on the s integrations may be approximated by the appropriate $\overline{s}$'s. Thus the resulting expression is

$$\overline{B(R)B(R+\Delta R)} = \int_0^{\overline{S(R)}} \int_0^{\overline{S(R+\Delta R)}} \frac{G_1 G_2}{R^2 (R+\Delta R)^2} \int_0^{D_{\xi 1}} \int_0^{D_{\xi 2}} \int_0^{D_{\gamma 1}} \int_0^{D_{\gamma 2}} k_1 k_2$$

$$\cot(\vec{n}_1, \vec{R}) \cot(\vec{n}_2, \overrightarrow{R+\Delta R}) \frac{P(\tan\gamma_1, \tan\gamma_2, \tan\xi_1, \tan\xi_2)}{P(\tan\gamma_1)P(\tan\gamma_2)P(\tan\xi_1)P(\tan\xi_2)}$$

$$\sqrt{\tan\gamma_1 - \tan\delta_1)(\tan\gamma_2 - \tan\delta_2)} \sqrt{\tan\xi_1 \tan\xi_2}\, dQ_{\gamma 1} dQ_{\gamma 2} dQ_{\xi 1} dQ_{\xi 2} ds_1\, ds_2$$

The Monte Carlo estimate of this, for N selections in the six dimensional space, where ϕ_1 and ϕ_2 are chosen from uniform distributions for 0 to 2π, is

$$\overline{B(R)B(R+\Delta R)} = \sum_{i=1}^{N} \frac{G_1 G_2 D_{\xi 1} D_{\xi 2} D_{\gamma 1} D_{\gamma 2}\, \overline{S(R)S(R+\Delta R)}}{R^2 (R+\Delta R)^2} k_1\, k_2 \cot(\vec{n}_1, \vec{R})$$

$$\cot(\vec{n}_2, \overrightarrow{R+\Delta R}) \frac{P(\tan\gamma_1, \tan\gamma_2, \tan\xi_1, \tan\xi_2)}{P(\tan\gamma_1)\, P(\tan\gamma_2)\, P(\tan\xi_1)\, P(\tan\xi_2)}$$

$$(\tan\gamma_1 - \tan\delta_1)\ (\tan\gamma_2 - \tan\delta_2)\ \tan\xi_1\ \tan\xi_2$$

where an i subscript has been left off all the variables. The variance of $\overline{B(R)B(R+\Delta R)}$ is calculated from a formula analogous to the one for $\sigma^2 \overline{(B(R))}$. This time the equations represent four equations for the four possible combinations of k_1 and k_2, namely $k_{H1}\, k_{H2}$, $k_{H1}\, k_{V2}$, $k_{V1}\, k_{H2}$, and $k_{V1}\, k_{V2}$. The code is presently arranged to calculate $g(t)\, g(t + \Delta t)$ from Eq. (40), but this will probably have to be changed because of the high degree of cancellation involved in the integrand. The Taylor expansion of Eq. (39) would be used.

6.0 RESULTS AND CONCLUSIONS

6.1 Numerical Results

The tests of the envelope code were executed to correspond to a lunar experiment conducted with the 68-cm Millstone Hill Radar, as reported by Evans and Pettengill.[4] A 30-microsecond pulse was employed, and the antenna gain was considered to be 37 db for all points on the lunar surface. Runs were made for assumed standard deviations of the slopes of 0.05 (2.9°) and 0.10 (5.7°). The answers obtained will be discussed separately for the first time interval, when the signal is at least in part due to the pulse passing over the nearest point on the surface, and for the middle time interval, when the signal is due to a pulse whose leading and trailing edges are both in contact with the surface. The signal in the final time interval was too small to be experimentally observable. Also, "shadow effects" have been ignored in the analytical procedure and in the code, and these will be important on the limbs of the sphere, which are being examined at late times.

Although the shape of the signal was calculated during the first time period, the answer includes the assumption of a square transmitted pulse, and is quite dependent on that assumption. Data on the shape of the transmitted pulse and of the received signal during this time period was not contained in Ref. 3. Also the analysis of the shape of this part of the signal would be complicated by need to consider the receiver frequency response. Therefore, comparisons with experiment in this time interval were limited to consideration of loop loss (P_T/P_R). At the time the trailing edge of the transmitted pulse was incident on the moon, the experimental and computed results were as follows:

Experimental	210 db
Computer (σ = 0.05)	213 db
Computer (σ = 0.10)	227 db

The experimental loss was obtained from Fig. 9 of Ref. 3 and is for a 12-microsecond pulse, but the analysis indicates the result should be relatively independent of pulse length.

---

[4] Evans, J. V., and Pettengill, G. H. "The Scattering Behavior of the Moon at Wavelengths of 3.6, 68, and 784 Centimeters", Journal of Geophysical Research, Vol. 68, No. 2, January 15, 1963.

Because the analysis only includes the part of the signal due to $\overline{B(R)}$ and not due to the random component, it is not possible to conclude at this point that the surface appears to have a standard deviation of about 0.05. It is only possible to say that during this time interval, the part of the signal due to $\overline{B(R)}$ is a substantial part of the total signal. At least for this part of the signal, the loop loss is quite a sensitive indicator of the surface roughness. Application of the correlation code will demonstrate whether this is the only significant part of the signal during this time period.

For the middle time interval, it was found that the part of the signal due to $\overline{B(R)}$ was much smaller than that observed experimentally, and thus that the dominant part of the signal is due to the random component of B (R). This can perhaps be best illustrated by comparing with experiment the answers obtained assuming an infinite-bandwidth transmitter and receiver. For this case the condition that $\int_0^T f(t)\, dt = 0$ is abandoned, and Eq. (34) is used for this time interval also. Note that this makes the signal directly proportional to $\overline{h(t)}$. Normalizing these signals to the signal used to compute the loop gain, above, gives the results shown in Fig. 3. The computed polarized signals $\overline{g_H(t)}$ and the computed depolarized signals $\overline{g_V(t)}$ are shown for both surface σ's, along with a curve derived from Fig. 6 of Ref. 3. It is seen that even under these extreme assumptions the computed signals are appreciably too low. When Eq. (35) was used the answers were two orders of magnitude lower, and even in this case only a small fraction of the signals would actually be observed through a narrow-band receiver. Evidently the correlation code will be required to obtain useful information in this time interval.

The estimates of the standard deviation of the results for the polarized signal were about 10-20%. The standard deviations for the depolarized signal ranged from a nominal 10% at 0.05 milliseconds to over 300 percent at 8 milliseconds. These calculations required 12 minutes of IBM 7040 computer time for all the calculations associated with one surface probability distribution.

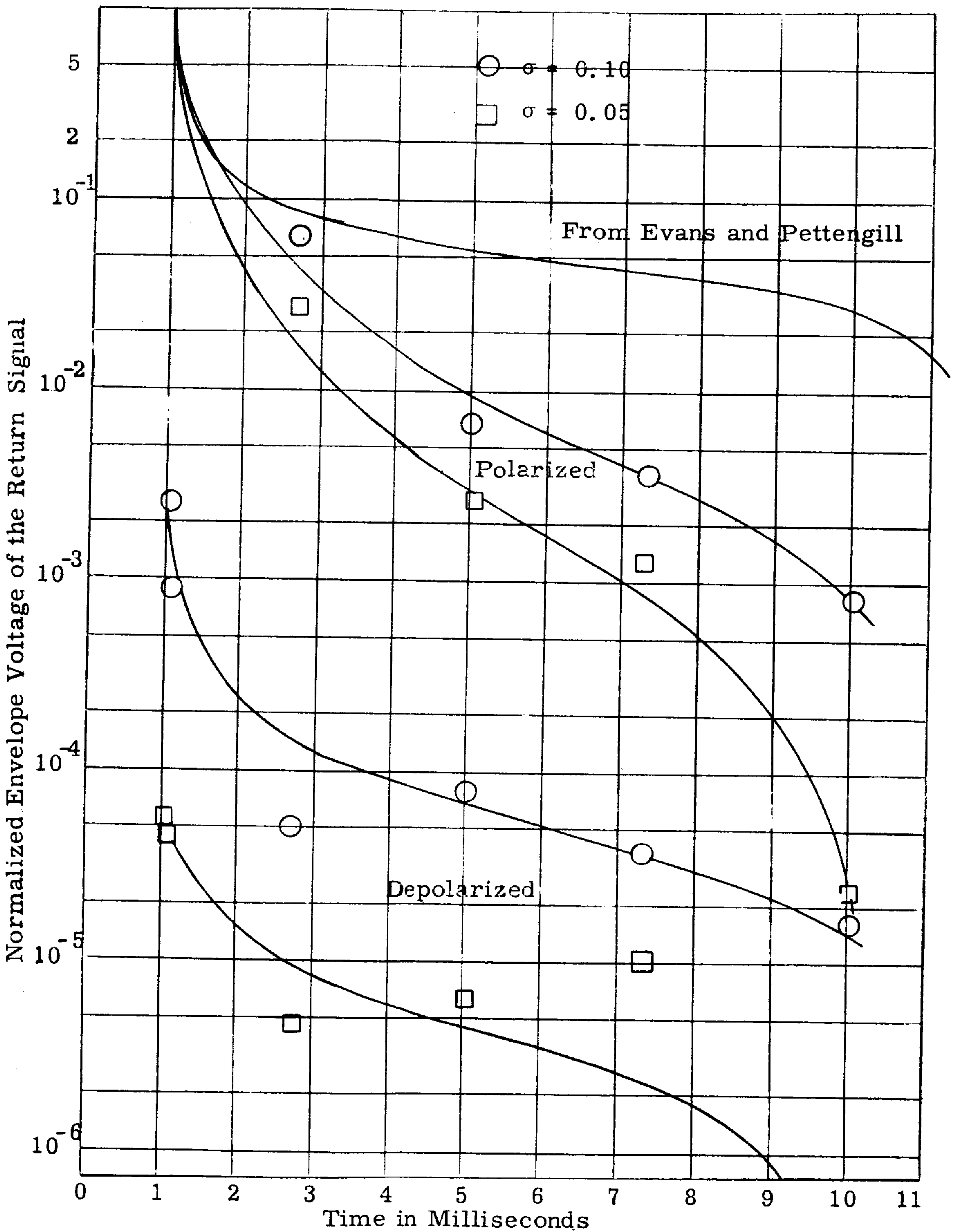

Fig. 3 - Computer Results

6.2 Conclusions and Recommendations for Future Work

The envelope code, in itself, is not an adequate tool for interpreting lunar radar return, but will be useful in conjunction with the correlation code. The correlation code should be modified to use a more sophisticated method of evaluating the return signal. Some additional analytical work may be necessary in this connection. However, the envelope code results indicate that a reasonably efficient method for evaluating measures of B(R) has been found. This is the main part of the correlation code, and it should not require anything but normal debugging. Future work should concentrate on using this code to evaluate the power versus time in the return signal. After satisfactory results are obtained for the power-time relationships, the autocorrelation function of the return signal can be explored.

It is recommended that initial efforts involve the lunar radar data. Once the lunar data has been interpreted, hypotheses concerning the Earth and other planets in our solar system may be examined. Finally, the program may be extended to include doppler effects to aid in the understanding of some additional problems.

APPENDIX I

Basore's Theory of The Impulse Response of a Reflecting Surface

The usual retarded vector potential version of Huygens Principle, after a two-fold application of Green's Theorem, can be expressed[1] as

$$A(t) = -\frac{1}{4\pi}\int_V \frac{1}{r}\left\{\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}\right\}\Bigg|_{t-r/c} dv \qquad \text{(I-1)}$$

$$+\frac{1}{4\pi}\int_S \frac{1}{r}\left[\frac{1}{c}\left(\frac{\partial A}{\partial t}\right)_{t-r/c} + \frac{A(t-r/c)}{r}\right]\cos(n,r) + \left(\frac{\partial A}{\partial n}\right)_{t-r/c}\Bigg\}\, da$$

Each component of the vector potential obeys such a relationship. For the present application, the closed surface S will be the expanding wave resulting from an impulse transmitted at $t - r/c - r_1/c$ where r_1 is the radius of the expanding shell. The potential will be computed at the origin of the impulse, so that at time t the volume within S is source free. The A (t) due to the reflection is designated A_2, and the potential due to the emitted impulse at radius r_1 from the origin is designated as A_1. Under these conditions, $A_2(t)$ is just the retarded version of $A_1(t)$ and

$$A_2(t) = \frac{1}{4\pi}\int_S \frac{1}{r}\left\{\left[\frac{1}{r}A_1 + \frac{1}{c}\dot{A}_1\right]\cos(n,r) + \frac{\partial A_1}{\partial r_1}\cos(n, r_1)\right\} da \qquad \text{(I-2)}$$

However, r is in the direction of the normal and cos(n, r)=1 and

[1] c.f. Slater and Frank "Electromagnetism," p. 170.

$$A_2(t) = \frac{1}{4\pi} \int_S \frac{1}{r} \left\{ \frac{1}{r} A_1 + \frac{1}{c} \dot{A}_1 + \frac{\partial}{\partial r} A_1 \right\} da \qquad \text{(I-3)}$$

If A_1 is in a homogeneous medium so that it can be described at every point by $\frac{A_o(t-r/c)}{r}$ then no reflection occurs and Eq. (3) gives $A(t)=0$. In the case of concern, a boundary intersects the spherical shell and A_1 is dependent upon the boundary conditions.

Now if A_1 is any function of r, but not of ϕ or θ, then for A_1 aligned with the z axis, $A_r = A_1 \cos\theta$, $A_\theta = -A_1 \sin\theta$, $A_\phi = 0$, and the r and θ components of curl A_1 are identically zero. Then

$$\text{curl } A_1 = \frac{1}{r} \left[\frac{\partial}{\partial r} (-r A_1 \sin\theta) - \frac{\partial A_1}{\partial \theta} \cos\theta \right] = \frac{\partial}{\partial r} A_\theta$$

Before the boundary is encountered the incident field is $H_1(t) = \frac{\partial}{\partial r} A_\theta$ which for $A_1 = A_o \frac{(t-r/c)}{r}$ is

$$H_1(t) = -\frac{1}{r} A_\theta - \frac{1}{c} \dot{A}_\theta \qquad \text{(I-4)}$$

At the boundary the actual field is

$$H_2(t) = \frac{\partial}{\partial r} A_\theta \qquad \text{(I-5)}$$

The θ component of A_2 is from Eq. (I-3)

$$A_{2\theta}(t) = \frac{1}{4\pi} \int_S \frac{1}{r} \left\{ \frac{1}{r} A_\theta + \frac{1}{c} \dot{A}_\theta + \frac{\partial}{\partial r} A_\theta \right\} da \qquad \text{(I-6)}$$

Substitution of Eqs. (I-4) and (I-5) into (I-6) gives

$$A_{2\theta}(t) = \frac{1}{4\pi} \int_S \frac{1}{r} \left[- H_1(t) + H_2(t) \right] \cdot \quad da$$

$$= \frac{1}{4\pi} \int_S \frac{H_3}{r} \cdot \quad da \qquad \text{(I-7)}$$

Assuming it is possible to interpret H_3 as the incident wave H_o times a reflection coefficient k, then

$$A(t) = \frac{1}{4\pi} \int_S \frac{k}{r} \; H_0 \; (t-r/c) \; da \qquad \text{(I-8)}$$

The magnitude of H_0 will depend upon the angle of incidence and the transmitted power. In terms of the usual radar symbols and a transmitted impulse function $P(t) = P_0 g^2(t)$

$$H_0(t-r/c) = \frac{1}{R} \sqrt{\frac{GP_0}{4\pi\eta}} \; \cos\,(n, R)\; g(t-R/c) \qquad \text{(I-9)}$$

Of the terms in Eq. (9), only G will vary as a function of position. Substitution of Eq. (I-9) into (I-8) yields, back at the transmitter,

$$A(t) = \frac{1}{4\pi R^2} \sqrt{\frac{P_0}{4\pi\eta}} \int_S \; k\sqrt{G} \quad \cos\,(n, R) g(t-\frac{2R}{c}) \; da \qquad \text{(I-10)}$$

and the surface S is the surface of intersection of the impulse and the boundary. If the duration of the impulse is Δu seconds, it will have a duration of $c\Delta u$ in distance units. Since the vector R and the normal n are not, in general, parallel, the area increment is

$$da = \frac{\Delta R ds}{\sin(n, R)} \tag{I-11}$$

where $\Delta R = c\Delta u$. When this is included, the expression for the magnitude of the vector potential becomes

$$A(t) = \frac{1}{4\pi R^2} \sqrt{\frac{P_0}{4\pi\eta}} \int_S k\sqrt{G} \quad g(t-2R/c) \cot(n, R) \, \Delta R ds \tag{I-12}$$

When g(t) is treated as an impulse, integration with respect to R yields the result

$$A(t) = \frac{1}{4\pi R^2} \left(\frac{c}{2}\right) \sqrt{\frac{P_0}{4\pi\eta}} \int_C k\sqrt{G} \quad \cot(n, R) ds \tag{I-13}$$

The c/2 multiplier arises from the conversion of a delta function in time to a delta function in range. The magnetic field can be obtained by treating A(t) as a function of R and computing the curl. The curl of a vector A aligned with the z axis is numerically equal to $\frac{\partial}{\partial r} A_\theta$, and is aligned in the ϕ direction. This implicitly states that each repeated wavelet arrives back at the transmitting antenna polarized perpendicular to the direction to the point at which it is reflected. The net effect of the simultaneous arrival from different points on the curve C can be included by introducing the cross section of a receiving antenna $G\lambda^2/4\pi$. The equivalent vector potential is

$$A'(R) = \frac{\lambda}{(4\pi R)^2} \frac{c}{2} \sqrt{\frac{P_0}{\eta}} \int_C kG \cot(n, R) ds \tag{I-14}$$

and the equivalent field is

$$H'(R) = \frac{\partial}{\partial R} \left\{ \frac{\lambda}{(4\pi R)^2} \quad \frac{c}{2} \sqrt{\frac{P_0}{\eta}} \int_C kG \cot(n, R) ds \right\} \tag{I-15}$$

The relative response is then

$$h(t) = \frac{\eta}{P_0} H'(R) \bigg|_{R = \frac{ct}{2}}$$

or

$$h(t) = \frac{\lambda}{(4\pi)^2} \frac{c}{2} \left\{ \frac{\partial}{\partial R} \left[\frac{1}{R^2} \int_C kG \cot(n, R)\, ds \right] \right\}_{R = \frac{ct}{2}} \qquad \text{(I-16)}$$

Finally, to obtain the result in the form that will be used, the partial derivative with respect to R is converted to one with respect to t, and

$$h(t) = \frac{\lambda}{(4\pi)^2} \left\{ \frac{\partial}{\partial t} \left[\frac{1}{R^2} \int kG \cot(n, R) ds \right]_{R = \frac{ct}{2}} \right\} \qquad \text{(I-17)}$$

Using Eq. (I-17) one can, for example, easily obtain the standard radar cross section of a conducting sphere and a conducting flat plate.

APPENDIX II

CODES

(a) The Envelope Code

```
C      TAN(XI)=ITH VALUE OF TAN(XI)
C      P(I)=ITH VALUE OF THE PROBABILTY DENSITY FUNTION
C      XICD(I)=ITH VALUE OF THE CUMULATIVE DENSITY FUNTION FOR XI
C      SIG(I)=ITH SIGMA VALUE(INPUT)
C      WT(I)=ITH WEIGHTING FACTOR(INPUT)
C      TANGM(I)=ITH VALUE OF TAN(GAMMA)
C      GMCD(I)=ITH VALUE OF THE CUMULATIVE DENSITY FUNCTION FOR GAMMA
C      TME=CURRENT TIME ENTRY
C      TIME(I)=ITH TIME VALUE FOR WHICH CALCULATIONS WERE PREFORMED
C      GTAV=VALUE FOR G(T) AVERAGE
C      GSIG=ERROR SIGMA FOR G(T) AVERAGE
C      XO=INITIAL AND FINAL VALUE(- AND +) FOR EITHER TAN(XI) OR
C      TAN(GAMMA)
C      DX=RUNNING INCREMENTAL VALUE OF X WHICH STANDS FOR EITHER TAN(XI)
C      OR TAN(GAMMA)
C      X=CURRENT VALUE FOR EITHER TAN(XI) OR TAN(GAMMA)
C      C=ACCUMULATION VARIABLE USED IN THE CDF CALCULATIONS FOR TAN(XI)
C      AND TAN(GAMMA)
C      Q=TOTAL ACCUMULATION VARIABLE USED IN THE CDF CALCULATION FOR
C      TAN(XI) AND TAN(GAMMA)
C      DX1=TEMPORARY STORAGE AREA FOR DX IN CASE THE FOLLOWING DX IS ZERO
C      N=NUMBER OF ENTRIES IN THE TAN(XI) CDF TABLE (AT THE END OF THE
C      SUBROUTINE ONLY)
C      DELTA=CONSTANT USED IN DETERMINING THE VALUE OF DX
C      NN=NUMBER OF ENTRIES IN THE TAN(GAMMA) CDF TABLE (AT THE END OF
C      THE SUBROUTINE ONLY)
C      D1=NORMALIZING FACTOR FOR THE TAN(XI) CDF
C      D2=NORMALIZING FACTOR FOR THE TAN(GAMMA) CDF
C      QK=CALCULATED CONSTANT USED TO CALCULATE THE INCREMENTAL VALUE OF
C      B
C      QMU1=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT(MU1)
C      QMU2=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT(MU2)
C      EPSI1=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT
C      EPSI2=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT
C      NRN=THE RANDOM NUMBER INITIALIZER(MUST BE BETWEEN 0 AND 500)
C      (INPUT)
C      I=SUBSCRIPTING VARIABLE ONLY
C      RNNO=CURRENT VALUE OF THE RANDOM NUMBER
```

```
C      RHO1=CALCULATED VALUE OF A CONSTANT USED IN DETERMINING QK
C      RHO2=CALCULATED VALUE OF A SECOND CONSTANT USED IN DETERMINING QK
C      NSIG=NUMBER OF SIGMAS BEING USED IN THE CURRENT PROBLEM(INPUT)
C      MUST BE 10 OR LESS
C      QN1=NUMBER OF X INCREMENTS USED IN CDF CALCULATION FOR TAN(XI)
C      MUST BE 250 OR LESS(INPUT)
C      DRG=RADAR TO CENTER OF OBJECTIVE DISTANCE IN KILOMETERS(INPUT)
C      ARD=CIRCULAR RADIUS OF OBJECTIVE IN KILOMETERS(INPUT)
C      QNMIN=MINIMUM NUMBER OF STEPS IN CIRCULAR ANULUS(INPUT)
C      QNRLM=NUMBER OF DESIRED RANDOM RANGES FOR PROBLEM(INPUT)
C      DPRLM=NUMBER OF RANDOM RANGES BETWEEN INTERMEDIATE PRINTOUTS
C      (INPUT)
C      QNINV=INVERSE OF THE MINIMUM NUMBER OF CIRCULAR STEPS
C      QNR=CURRENT NUMBER OF RANDOM RANGES
C      DRGSQ=SQUARE OF THE RADAR TO OBJECTIVE CENTER DISTANCE
C      ARDSQ=SQUARE OF THE OBJECTIVE RADIUS
C      RTDMA=SQUARE ROOT OF DRG SQUARED MINUS ARD SQUARED
C      DMA=DRG MINUS ARD
C      QN2=NUMBER OF X INCREMENTS USED IN CDF CALCULATION FOR TAN(GAMMA)
C      MUST BE 250 OR LESS(INPUT)
C      G=CONSTANT USED TO EVALUATE BRAV(INPUT)
C      T=RADAR PULSE WIDTH IN MILLISECONDS(INPUT)
C      QNT1=NUMBER OF DESIRED TIME INTERVALS BETWEEN TO AND T(INPUT)
C      QNT2=NUMBER OF DESIRED TIME INTERVALS BETWEEN TO+T AND TN+T(INPUT)
C      THE SUM OF QNT1 AND QNT2 MUST BE LESS THAN 100
C      OMEGA=ANGULAR FREQUENCY OF RADAR IN RADIANS PER MILLISECOND
C      (INPUT)
C      TO=INITIAL STARTING TIME AS DETERMINED BY THE RANGES INVOLVED
C      TOPT=TO PLUS T
C      TN=MAXIMUM RADAR RETURN TIME AS DETERMINED BY THE RANGES INVOLVED
C      TNPT=TN PLUS T
C      TINV=INVERSE OF T
C      DT=CURRENT TIME INCREMENT
C      DTA=TIME INCREMENT DETERMINED BY TO,TN,AND QNT2
C      TMUL=CURRENT VALUE OF MULTIPLIER USED IN CALCULATING F(T)
C      TAU=VARIABLE RELATING RANGE AND TIME
C      RNG=RANGE FROM THE RADAR TO THE TARGET
C      RNGSQ=SQUARE OF RNG
```

```
C     COSB=COSINE OF BETA
C     SEC2B=SQUARE OF SECANT OF BETA
C     TANB=TANGENT OF BETA
C     COST=COSINE OF THETA
C     SINT=SINE OF THETA
C     DS=INCREMENTAL ARC LENGTH
C     PVDSL=PHI OVER DS LIMIT
C     PHODS=PHI OVER DS
C     B=SUMMING VARIABLE USED IN DETERMINING BRAV
C     QNB=CURRENT NUMBER OF B INTERVALS
C     J=INTERMEDIATE SUBSCRIPTING VARIABLE
C     TNXI=CURRENT RANDOMLY SELECTED VALUE OF TAN(XI)
C     TNGM=CURRENT RANDOMLY SELECTED VALUE OF TAN(GAMMA)
C     TNBMG=TAN(BETA) MINUS TAN(GAMMA)
C     RDINV=INVERSE OF A RADICAL USED IN EVALUATING PHODS
C     ACTNR=VALUE OF COTANGENT OF THE ANGLE BETWEEN N AND R
C     SRAV=AVERAGE ROUGH ANULUS ARC LENGTH
C     BRAV=AVERAGE VALUE OF THE INTERMEDIATE INTEGRAL(BETA AVERAGE)
C     BR2AV=SUM OF SQUARES OF BRAV
C     D1BAV=VALUE OF FIRST DERIVATIVE OF BRAV
C     D1BR2=SUM OF SQUARES OF FIRST DERIVATIVE OF BRAV
C     D2BAV=VALUE OF SECOND DERIVATIVE OF BRAV
C     D2BR2=SUM OF SQUARES OF SECOND DERIVATIVE OF BRAV
C     SENSE SWITCH 1 DOWN GIVES TAN(XI) CDF LIST OUT
C     SENSE SWITCH 2 DOWN GIVES TAN(GAMMA) CDF LIST OUT
C     SENSE SWITCH 3 DOWN GIVES K CONSTANTS LIST OUT
C     SENSE SWITCH 4 DOWN GIVES A PAUSE IN THE PROGRAM
C     SENSE SWITCH 5 DOWN GIVES THE RANGE AND S(R) AVERAGE PRINT OUT
C     MAIN PROGRAM FOR MONTE CARLO EVALUATION OF AN INTERGL 25 JULY 1963
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500),GM
     1CD(500),B(2),QK(2),BRAV(2,100),BR2AV(2,100),TIME(100),D1BAV(2,100)
     2,D1BR2(2,100),D2BAV(2,100),D2BR2(2,100)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK
39    READ 8,QMU1,QMU2,EPSI1,EPSI2,NRN
8     FORMAT(4E13.8,I5)
      IF(NRN-500)16,16,17
17    PRINT 14,NRN
```

```
14    FORMAT(14HOERROR AS NRN=I5)
      STOP
16    DO 15 I=1,NRN
15    RNNO=RAN1F(-1)
      DO 110 I=1,100
      DO 110 J=1,2
      BRAV(J,I)=0.0
110   BR2AV(J,I)=0.0
      RHO1=SQRTF(QMU2*EPSI1/(EPSI2*QMU1))
      RHO2=SQRTF(QMU1*EPSI1/(QMU2*EPSI2))
      READ 1,NSIG,QN1,DRG,ARD,QNMIN,QNRLM,DPRLM
1     FORMAT(I5,F5.0,5E13.8)
      IF(NSIG-10)20,20,21
21    PRINT 3,NSIG
3     FORMAT(12HOERROR NSIG=I5)
      STOP
20    IF(QN1-250.0)22,22,23
23    PRINT 4,QN1
4     FORMAT(10HOERROR N1=E15.8)
      STOP
22    READ 2,(SIG(I),WT(I),I=1,NSIG)
2     FORMAT(6E13.8)
      QNINV=1.0/QNMIN
      CALL CDFXI(QN1,NSIG)
      DRGSQ=DRG*DRG
      ARDSQ=ARD*ARD
      RTDMA=SQRTF(DRGSQ-ARDSQ)
      DMA=DRG-ARD
      READ 5,QN2,G,T,QNT1,QNT2,OMEGA
5     FORMAT(F5.0,5E13.6)
      IF(QN2-250.0)32,32,33
33    PRINT 6,QN2
6     FORMAT(10HOERROR N2=E15.8)
      STOP
32    TO=0.0066666667*DMA
      TOPT=TO+T
      PRINT 200,QMU1,QMU2,EPSI1,EPSI2,DRG,ARD,G,T,OMEGA,(SIG(I),WT(I),I=
     11,NSIG)
```

```
200   FORMAT(7H1MU(1)=E15.8,3X,6HMU(2)=E15.8,3X,11HEPSILON(1)=E15.8,3X,1
     111HEPSILON(2)=E15.8/10H DISTANCE=E15.8,3X,7HRADIUS=E15.8,3X,5HGAIN=
     2E15.8,3X,12HPULSE WIDTH=E15.8/33H ANGULAR FREQUENCY(RADIANS/MSEC)=
     3E15.8/7X,5HSIGMA,12X,6HWEIGHT/(1H E15.8,E18.8))
      NTME=1
      TN=0.0066666667*RTDMA
      TNPT=TN+T
      TINV=1.0/T
      DT=T/QNT1
      DTA=(TN-TO)/QNT2
      TIME(NTME)=TO+DT
35    QNR=0.0
      IF(TN-TIME(NTME))40,40,41
40    TMUL=TNPT-TIME(NTME)
      GO TO 30
41    IF(TIME(NTME)-TOPT)42,43,43
42    TMUL=TIME(NTME)-TO
      GO TO 30
43    DT=DTA
      TMUL=T
30    TAU=TIME(NTME)-TMUL*RAN1F(-1)
      RNG=150.0*TAU
      RNGSQ=RNG*RNG
      COSB=(RNGSQ+ARDSQ-DRGSQ)*0.5/(RNG*ARD)
      SEC2B=1.0/(COSB*COSB)
      TANB=SQRTF(SEC2B-1.0)
      COST=(ARD-RNG*COSB)/DRG
      SINT=SQRTF(1.0-COST*COST)
      DS=6.2831853*ARD*SINT*QNINV
      IF(TANB-SIG(1)*2.828427)230,231,231
230   DS=DS*SIG(1)*2.828427/TANB
231   PVDSL=6.2831853/DS
      QNR=QNR+1.0
      CALL CDFGM(QN2,NSIG,TANB)
C     CONTINUATION OF MAIN PROGRAM
      PHODS=0.0
      B(1)=0.0
      B(2)=0.0
```

```
      QNB=0.0
70    QNB=QNB+1.0
      RNNO=RAN1F(-1)
      DO 60 I=1,N
      IF(RNNO-XICD(I))61,62,60
60    CONTINUE
      PRINT 7,XICD(I),RNNO
7     FORMAT(30HOERROR AS MAX VALUE OF XI CDF=E13.6,29H AND RANDOM NUMBE
     1R GENERATED=E15.8)
      STOP
61    J=I-1
      TNXI=(RNNO-XICD(I))*(TANXI(J)-TANXI(I))/(XICD(J)-XICD(I))+TANXI(I)
      GO TO 63
62    TNXI=TANXI(I)
63    RNNO=RAN1F(-1)
      DO 65 I=1,NN
      IF(RNNO-GMCD(I))66,67,65
65    CONTINUE
      PRINT 11,GMCD(I),RNNO
11    FORMAT(30HOERROR AS MAX VALUE OF GM CDF=E13.6,29H AND RANDOM NUMBE
     1R GENERATED=E15.8)
      STOP
66    J=I-1
      TNGM=(RNNO-GMCD(I))*(TANGM(J)-TANGM(I))/(GMCD(J)-GMCD(I))+TANGM(I)
      GO TO 69
67    TNGM=TANGM(I)
69    TNBMG=TANB+TNGM
      RDINV=1.0/SQRTF(TNBMG*TNBMG+TNXI*TNXI*SEC2B)
      ACTNR=(1.0-TANB*TNGM)*RDINV
      CALL CONSK(TNXI,ACTNR,RHO1,RHO2,PHODS,DS)
      Q=SQRTF(ABSF(TNBMG*TNXI))*ACTNR
      DO 102 I=1,2
102   B(I)=B(I)+QK(I)*Q
      IF(SENSE SWITCH 4)260,261
260   PRINT 270
270   FORMAT(1H1)
      PAUSE
261   PHODS=PHODS+TNBMG*RDINV/(ARD*SINT)
```

```
      IF(PHODS-PVDSL)70,71,71
71    SRAV=QNB*DS
      DO 100 I=1,2
      DB=G*DS*B(I)/(RNGSQ*D1*D2)
      BRAV(I,NTME)=BRAV(I,NTME)+DB
100   BR2AV(I,NTME)=BR2AV(I,NTME)+DB*DB
      IF(SENSE SWITCH 5)240,250
240   PRINT 19,RNG,SRAV
19    FORMAT(3H R=E15.8,5X,13HAVE-AGE S(R)=E15.8)
      LINE=LINE+1
      IF(LINE-52)250,250,125
125   LINE=0
      PRINT 270
250   IF(QNR-QNRLM)30,84,84
84    NTME=NTME+1
      TIME(NTME)=TIME(NTME-1)+DT
      IF(TIME(NTME)-TNPT)35,35,85
85    NTME=NTME-1
      KGO=1
      DO 111 I=1,2
      T1M=TIME(1)-TO
      T2M=TNPT-TIME(NTME-1)
      D1BAV(I,1)=BRAV(I,1)/T1M
      D1BAV(I,NTME)=-BRAV(I,NTME)/T2M
      D1BR2(I,1)=BR2AV(I,1)/T1M
      D1BR2(I,NTME)=BR2AV(I,NTME)/T2M
      LTME=NTME-1
      DO 111 J=2,LTME
      T1M=TIME(J)-TIME(J-1)
      T2M=TIME(J+1)-TIME(J)
      D1BAV(I,J)=(BRAV(I,J)-BRAV(I,J-1))/T1M
111   D1BR2(I,J)=(BR2AV(I,J)+BR2AV(I,J-1))/T1M
      GCONS=37.5/(OMEGA*OMEGA*QNR)
      DO 140 I=1,2
      DO 140 J=1,NTME
117   GTAV=GCONS*(0.6366198*ABSF(D1BAV(I,J)))
      GFAC=(0.6366198*ABSF(D1BR2(I,J)))*GCONS*GCONS
120   TME=TIME(J)-TO
```

```
      GO TO (121,122),I
121   PRINT 9,QNR,TME,GTAV,GSIG
9     FORMAT(7HO RANGE,7X,4HTIME,6X,12HAVERAGE G(T),5X,13H(ERROR SIGMA)/
     17H NUMBER,4X,10H(MILLISEC),5X,8HPARALLEL/1H F4.0,3X,E13.6,E14.6,5X
     2,1H(E13.6,1H))
      GO TO 112
122   GO TO (123,124),KGO
123   PRINT 200,QMU1,QMU2,EPSI1,EPSI2,DRG,ARD,G,T,OMEGA,(SIG(I),WT(I),I=
     11,NSIG)
      KGO=2
      LINE=4+NSIG
124   PRINT 10,QNR,TME,GTAV,GSIG
10    FORMAT(7HO RANGE,7X,4HTIME,6X,12HAVERAGE G(T),5X,13H(ERROR SIGMA)/
     17H NUMBER,4X,10H(MILLISEC),2X,13HPERPENDICULAR/1H F4.0,3X,E13.6,E1
     24.6,5X,1H(E13.6,1H))
112   LINE=LINE+4
      IF(LINE-52)140,140,133
133   LINE=0
      PRINT 134
134   FORMAT(1H1)
140   CONTINUE
      PRINT 135
135   FORMAT(15HOEND OF PROBLEM)
      GO TO 39
      END
C     COMPUTATION OF CDF FOR TAN(XI) WITH BIASING
      SUBROUTINE CDFXI(QN1,NSIG)
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500),GM
     1CD(500),QK(2)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK
      XO=3.0*SIG(1)
      DELTA=4.0*SQRTF(XO)/QN1
      DX=DELTA*SQRTF(XO)
      X=-XO-DX
      TANXI(1)=X
      P(1)=0.0
      N=1
```

```
      Q=0.0
      XICD(1)=0.0
19    N=N+1
      C=0.0
      X=X+DX
      DO 20 I=1,NSIG
      CSIG=SIG(I)
20    C=C+0.3989423*EXPF(-X*X*0.5/(CSIG*CSIG))*WT(I)/CSIG
      Q=Q+C
      DX1=DX
      XICD(N)=Q
      TANXI(N)=X
      P(N)=C
      DX=DELTA*SQRTF(ABSF(X))
      IF(DX)24,25,26
25    DX=DX1
26    IF(X-XO)19,21,21
24    PRINT 10,DX
10    FORMAT(21HOPROGRAM ERROR AS DX=E11.4)
      STOP
21    P(N)=0.0
      D1=1.0/Q
      DO 30 I=2,N
30    XICD(I)=XICD(I)*D1
      IF(SENSE SWITCH 1)31,32
31    PRINT 1,QN1,NSIG,(TANXI(I),XICD(I),P(I),I=1,N)
1     FORMAT(29H1NUMBER OF TAN(XI) DIVISIONS=F6.1,32H AND THE NUMBER OF
     1INPUT SIGMAS=I5/5X,7HTAN(XI),8X,18HCUMULATIVE DENSITY,6X,19HGENERA
     2L PROBABILITY/19X,20HFUNCTION FOR TAN(XI),7X,16HDENSITY FUNCTION/(
     3E14.6,8X,E13.6,12X,E13.6))
32    CONTINUE
      RETURN
      END
C     COMPUTATION OF CDF FOR TAN(GAMMA) WITH BIASING
      SUBROUTINE CDFGM(QN2,NSIG,TANB)
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500),GM
     1CD(500),QK(2)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
```

```
      11,D2,QK
       XOA=3.0*SIG(1)
       SQRTN=SQRTF(TANB+XOA)
       DELTA=2.0*(SQRTF(ABSF(TANB-XOA))+SQRTN)/QN2
       DX=DELTA*SQRTN
       X=-XOA-DX
       NN=1
       GMCD(NN)=0.0
       Q=0.0
       TANGM(NN)=X
       J=2
39     X=X+DX
       DX1=DX
       DX=DELTA*SQRTF(ABSF(TANB+X))
       IF(DX)52,50,51
50     DX=DX1
51     IF(X-TANXI(1))40,40,41
41     IF(X-TANXI(N))42,43,43
42     DO 45 I=J,N
       IF(X-TANXI(I))46,47,45
45     CONTINUE
46     J=I-1
       C=(X-TANXI(J))*(P(I)-P(J))/(TANXI(I)-TANXI(J))+P(J)
       J=I
       GO TO 48
52     PRINT 1,DX
1      FORMAT(34HOPROGRAM ERROR IN GAMMA CDF AS DX=E11.4)
       STOP
40     C=0.0
       GO TO 48
47     J=I
       C=P(I)
48     Q=Q+C
       NN=NN+1
       GMCD(NN)=Q
       TANGM(NN)=X
       IF(X-XOA)39,43,43
43     D2=1.0/Q
```

```
      DO 55 I=1,NN
55    GMCD(I)=GMCD(I)*D2
      IF(SENSE SWITCH 2)31,32
31    PRINT 2,QN2,NSIG,TANB,(TANGM(I),GMCD(I),I=1,NN)
2     FORMAT(32H1NUMBER OF TAN(GAMMA) DIVISIONS=F6.1,25H, NUMBER OF INPU
     1T SIGMAS=I5,15H AND TAN(BETA)=E13.6/3X,10HTAN(GAMMA),9X,18HCUMULAT
     2IVE DENSITY/20X,23HFUNCTION FOR TAN(GAMMA)/(E14.6,10X,E13.6))
32    CONTINUE
      RETURN
      END
C     COMPUTATION OF THE CONSTANT K
      SUBROUTINE CONSK(TNXI,ACTNR,RHO1,RHO2,PHODS,DS)
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500),GM
     1CD(500),QK(2)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK
      SN2NR=1.0/(ACTNR*ACTNR+1.0)
      COSNR=SQRTF(1.0-SN2NR)
      PHI=PHODS*DS
      XI=ATANF(TNXI)
      PHMXI=PHI-XI
      RADCL=SQRTF(1.0-RHO2*RHO2*SN2NR)
      QK1=(COSNR-RHO1*RADCL)/(COSNR+RHO1*RADCL)
      QK2=(RADCL-RHO1*COSNR)/(RADCL+RHO1*COSNR)
      COSPX=COSF(PHMXI)
      QK(2)=(QK1-QK2)*SINF(PHMXI)*COSPX
      COSPX=COSPX*COSPX
      QK(1)=QK1*COSPX+QK2*(1.0-COSPX)
      IF(SENSE SWITCH 3)31,32
31    PRINT 1,TNXI,ACTNR,RHO1,RHO2,PHODS,DS,QK(1),QK(2)
1     FORMAT(9HOTAN(XI)=E13.6,3X,27HABSOLUTE VALUE OF COT(N,R)=E13.6,3X,
     17HRHO(1)=E13.6,3X,7HRHO(2)=E13.6/8H PHI/DS=E13.6,3X,3HDS=E13.6,3X,
     212HK(PARALLEL)=E13.6,3X,17HK(PERPENDICULAR)=E13.6)
32    CONTINUE
      RETURN
      END
```

(b) The Correlation Code

```
C      P(I)=ITH VALUE OF THE PROBABILTY DENSITY FUNCTION
C      XICD(I)=ITH VALUE OF THE CUMULATIVE DENSITY FUNTION FOR XI
C      SIG(I)=ITH SIGMA VALUE(INPUT)
C      WT(I)=ITH WEIGHTING FACTOR(INPUT)
C      TANGM(I)=ITH VALUE OF TAN(GAMMA)
C      GMCD(I)=ITH VALUE OF THE CUMULATIVE DENSITY FUNCTION FOR GAMMA
C      TME=CURRENT TIME ENTRY
C      TIME=TIME VALUE FOR WHICH CALCULATIONS WERE PREFORMED
C      GTAV=VALUE FOR G(T) AVERAGE
C      GSIG=TIME SIGMA ASSOCIATED WITH PULSE DELAY
C      DSIG=INCREMENTAL VALUE OF GSIG(INPUT)
C      XO=INITIAL AND FINAL VALUE(- AND +) FOR EITHER TAN(XI) OR
C      TAN(GAMMA)
C      DX=RUNNING INCREMENTAL VALUE OF X WHICH STANDS FOR EITHER TAN(XI)
C      OR TAN(GAMMA)
C      X=CURRENT VALUE FOR EITHER TAN(XI) OR TAN(GAMMA)
C      C=ACCUMULATION VARIABLE USED IN THE CDF CALCULATIONS FOR TAN(XI)
C      AND TAN(GAMMA)
C      Q=TOTAL ACCUMULATION VARIABLE USED IN THE CDF CALCULATION FOR
C      TAN(XI) AND TAN(GAMMA)
C      DX1=TEMPORARY STORAGE AREA FOR DX IN CASE THE FOLLOWING DX IS ZERO
C      N=NUMBER OF ENTRIES IN THE TAN(XI) CDF TABLE (AT THE END OF THE
C      SUBROUTINE ONLY)
C      DELTA=CONSTANT USED IN DETERMINING THE VALUE OF DX
C      NN=NUMBER OF ENTRIES IN THE TAN(GAMMA) CDF TABLE (AT THE END OF
C      THE SUBROUTINE ONLY)
C      D1=NORMALIZING FACTOR FOR THE TAN(XI) CDF
C      D2=NORMALIZING FACTOR FOR THE TAN(GAMMA) CDF
C      QK=CALCULATED CONSTANT USED TO CALCULATE THE INCREMENTAL VALUE OF
C      B
C      QMU1=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT(MU1)
C      QMU2=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT(MU2)
C      EPSI1=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT
C      EPSI2=INPUT PARAMETER CHARACTERISTIC OF THE MATERIALS PRESENT
C      NRN=THE RANDOM NUMBER INITIALIZER(MUST BE BETWEEN 0 AND 500)
C      (INPUT)
C      I=SUBSCRIPTING VARIABLE ONLY
C      RNNO=CURRENT VALUE OF THE RANDOM NUMBER
```

```
C      RHO1=CALCULATED VALUE OF A CONSTANT USED IN DETERMINING QK
C      RHO2=CALCULATED VALUE OF A SECOND CONSTANT USED IN DETERMINING QK
C      NSIG=NUMBER OF SIGMAS BEING USED IN THE CURRENT PROBLEM(INPUT)
C      MUST BE 10 OR LESS
C      QN1=NUMBER OF X INCREMENTS USED IN CDF CALCULATION FOR TAN(XI)
C      MUST BE 250 OR LESS(INPUT)
C      DRG=RADAR TO CENTER OF OBJECTIVE DISTANCE IN KILOMETERS(INPUT)
C      ARD=CIRCULAR RADIUS OF OBJECTIVE IN KILOMETERS(INPUT)
C      QNRLM=NUMBER OF DESIRED RANDOM RANGES FOR PROBLEM(INPUT)
C      DPRLM=NUMBER OF RANDOM RANGES BETWEEN INTERMEDIATE PRINTOUTS
C      (INPUT)
C      QNR=CURRENT NUMBER OF RANDOM RANGES
C      DRGSQ=SQUARE OF THE RADAR TO OBJECTIVE CENTER DISTANCE
C      ARDSQ=SQUARE OF THE OBJECTIVE RADIUS
C      RTDMA=SQUARE ROOT OF DRG SQUARED MINUS ARD SQUARED
C      DMA=DRG MINUS ARD
C      QN2=NUMBER OF X INCREMENTS USED IN CDF CALCULATION FOR TAN(GAMMA)
C      MUST BE 250 OR LESS(INPUT)
C      G=CONSTANT USED TO EVALUATE GCONS(INPUT)
C      T=RADAR PULSE WIDTH IN MILLISECONDS(INPUT)
C      QNT1=NUMBER OF DESIRED TIME INTERVALS BETWEEN TO AND TO+T(INPUT)
C      QNT2=NUMBER OF DESIRED TIME INTERVALS BETWEEN TO+T AND TN+T(INPUT)
C      THE SUM OF QNT1 AND QNT2 MUST BE LESS THAN 100
C      OMEGA=ANGULAR FREQUENCY OF RADAR IN RADIANS PER MILLISECOND
C      (INPUT)
C      TO=INITIAL STARTING TIME AS DETERMINED BY THE RANGES INVOLVED
C      TOPT=TO PLUS T
C      TN=MAXIMUM RADAR RETURN TIME AS DETERMINED BY THE RANGES INVOLVED
C      TNPT=TN PLUS T
C      DT=CURRENT TIME INCREMENT
C      DTA=TIME INCREMENT DETERMINED BY TO,TN,AND QNT2
C      TMUL=CURRENT VALUE OF MULTIPLIER USED IN CALCULATING F(T)
C      TAU=VARIABLE RELATING RANGE AND TIME
C      TMU=VARIABLE RELATING RANGE AND TIME
C      RNG1=RANGE FROM THE RADAR TO THE TARGET ASSOCIATED WITH TAU
C      RNG2=RANGE FROM RADAR TO TARGET ASSOCIATED WITH TMU
C      RG1SQ=SQUARE OF RNG1
C      RG2SQ=SQUARE OF RNG2
```

```
C     R12SQ=SQUARE OF RNG1 TIMES THE SQUARE OF RNG2
C     COSB1=COSINE OF BETA(1)
C     COSB2=COSINE OF BETA(2)
C     SC2B1=SQUARE OF SECANT OF BETA(1)
C     SC2B2=SQUARE OF SECANT OF BETA(2)
C     TANB1=TANGENT OF BETA(1)
C     TANB2=TANGENT OF BETA(2)
C     COST1=COSINE OF THETA(1)
C     COST2=COSINE OF THETA(2)
C     SINT1=SINE OF THETA(1)
C     SINT2=SINE OF THETA(2)
C     J=INTERMEDIATE SUBSCRIPTING VARIABLE
C     TNXI=CURRENT RANDOMLY SELECTED VALUE OF TAN(XI)
C     TNXI1=CURRENT RANDOM VALUE OF TAN(XI1)
C     TNXI2=CURRENT RANDOM VALUE OF TAN(XI2)
C     TNGM=CURRENT RANDOMLY SELECTED VALUE OF TAN(GAMMA)
C     TNGM1=CURRENT RANDOM VALUE OF TAN(GAMMA(1))
C     TNGM2=CURRENT RANDOM VALUE OF TAN(GAMMA(2))
C     TBMG1=TAN(BETA(1)) MINUS TAN(GAMMA(1))
C     TBMG2=TAN(BETA(2)) MINUS TAN(GAMMA(2))
C     RINV1=INVERSE OF A RADICAL USED IN EVALUATING ACNR1
C     RINV2=INVERSE OF A RADICAL USED IN EVALUATING ACNR2
C     SRAV=AVERAGE ROUGH ANULUS ARC LENGTH(INPUT)
C     SENSE SWITCH 1 DOWN GIVES TAN(XI) CDF LIST OUT
C     SENSE SWITCH 2 DOWN GIVES TAN(GAMMA) CDF LIST OUT
C     SENSE SWITCH 3 DOWN GIVES K CONSTANTS LIST OUT
C     SENSE SWITCH 4 DOWN GIVES THE BETA LIST OUT
C     SENSE SWITCH 5 DOWN GIVES THE W PRINT OUT
C     SENSE SWITCH 6 DOWN GIVES A PAUSE IN THE PROGRAM
C     MAIN PROGRAM MONTE CARLO INTEGRATION NUMBER 2 BY D. L. SUMMERS
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500,2),
     1GMCD(500,2),D2(2),ALFA(10),SRAV(500),R(500),ABR12(4),GTAV(4),DF2SM
     2(4),SSIG(4)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK,COST1,COST2,SINT1,SINT2,ALFA,W,NSIG,SRAV,R,ABR12
39    READ 8,QMU1,QMU2,EPSI1,EPSI2,NRN,DSIG,NP
8     FORMAT(4E13.8,I5,E13.8,I5)
      IF(NRN-500)16,16,17
```

```
17      PRINT 14,NRN
14      FORMAT(14HOERROR AS NRN=I5)
        STOP
16      IF(NP-10)82,82,83
83      PRINT 84,NP
84      FORMAT(39HOERROR AS MAX VALUE OF NP IS 10 AND NP=I5)
        STOP
82      READ 2,(ALFA(I),I=1,NP)
        READ 84,NS,(SRAV(I),R(I),I=1,NS)
84      FORMAT(I5,(6E13.8))
        DO 15 I=1,NRN
15      RNNO=RAN1F(-1)
        RHO1=SQRTF(QMU2*EPSI1/(EPSI2*QMU1))
        RHO2=SQRTF(QMU1*EPSI1/(QMU2*EPSI2))
        READ 1,NSIG,QN1,DRG,ARD,QNRLM,DPRLM
1       FORMAT(I5,F5.0,4E13.8)
        IF(NSIG-10)20,20,21
21      PRINT 3,NSIG
3       FORMAT(12HOERROR NSIG=I5)
        STOP
20      IF(QN1-250.0)22,22,23
23      PRINT 4,QN1
4       FORMAT(10HOERROR N1=E15.8)
        STOP
22      READ 2,(SIG(I),WT(I),I=1,NSIG)
2       FORMAT(6E13.8)
        CALL CDFXI(QN1,NSIG)
        DRSQ=DRG*DRG
        ARDSQ=ARD*ARD
        RTDMA=SQRTF(DRGSQ-ARDSQ)
        DMA=DRG-ARD
        READ 5,QN2,G,T,QNT1,QNT2,OMEGA
5       FORMAT(F5.0,5E13.8)
        IF(QN2-250.0)32,32,33
33      PRINT 6,QN2
6       FORMAT(10HOERROR N2=E15.8)
        STOP
32      PRINT 100,QMU1,QMU2,EPSI1,EPSI2,DRG,ARD,G,T,OMEGA,(SIG(I),WT(I),I=
```

```
     11,NSIG)
100   FORMAT(7H1MU(1)=E15.8,3X,6HMU(2)=E15.8,3X,11HEPSILON(1)=E15.8,3X,1
     11HEPSILON(2)=E15.8/10H DISTANCE=E15.8,3X,7HRADIUS=E15.8,3X,5HGAIN=
     2E15.8,3X,12HPULSE WIDTH=E15.8/37H ANGULAR FREQUENCY(RADIANS/MILLIS
     3EC)=E15.8/7X,5HSIGMA,12X,6HWEIGHT/1H E15.8,E18.8)
      PRINT 101,(ALFA(I),I=1,NP)
101   FORMAT(7X,5HALPHA/1H E15.8)
      LINE=5+NSIG+NP
      GSIG=0.0
      TO=0.0066666667*DMA
      TOPT=TO+T
      TN=0.0066666667*RTDMA
      TNPT=TN+T
      DTA=(TN-TO)/QNT2
96    DT=T/QNT1
      TIME=TO+DT
94    QNR=0.0
      PRTLM=0.0
      IPRT=1
      DO 85 I=1,4
      F(I)=0.0
85    DF2SM(I)=0.0
92    PRTLM=PRTLM+DPRLM
86    QNR=QNR+1
      IF(SENSE SWITCH 6)300,301
300   PRINT 104
      PAUSE
301   IF(GSIG-T)40,41,42
42    IF(GSIG-TN+TOPT)41,43,44
44    IF(GSIG-TN+TO)43,59,59
40    IF(TIME-TOPT+GSIG)46,47,48
48    IF(TIME-TOPT)47,49,50
50    IF(TIME-TN+GSIG)49,51,52
52    IF(TIME-TN)51,53,53
41    IF(TIME-TOPT)47,49,56
56    IF(TIME-TN+GSIG)49,51,51
43    IF(TIME-TN+GSIG)47,59,60
60    IF(TIME-TOPT)59,61,61
```

```
47      TMUL2=T
        GO TO 62
49      TMUL1=T
        DT=DTA
        GO TO 63
59      TMUL1=TIME-TO
51      ISGO=1
66      TMUL2=TNPT-TIME-GSIG
        TMU=TN-TMUL2*RAN1F(-1)
        GO TO(64,65),ISGO
53      TMUL1=TNPT-TIME
        TAU=TN-TMUL1*RAN1F(-1)
        ISGO=2
        GO TO 66
61      TMUL1=T
        GO TO 51
46      TMUL2=TIME+GSIG-TO
62      TMUL1=TIME-TO
63      TMU=TIME+GSIG-TMUL2*RAN1F(-1)
64      TAU=TIME-TMUL1*RAN1F(-1)
65      RNG1=150.0*TAU
        RNG2=150.0*TMU
        PHI1=6.2831853*RAN1F(-1)
        PHI2=6.2831853*RAN1F(-1)
        DPHI=PHI1-PHI2
        RG1SQ=RNG1*RNG1
        RG2SQ=RNG2*RNG2
        R12SQ=RG1SQ*RG2SQ
        COSB1=(RG1SQ+ARDSQ-DRGSQ)*0.5/(RNG1*ARD)
        COSB2=(RG2SQ+ARDSQ-DRGSQ)*0.5/(RNG2*ARD)
        SC2B1=1.0/(COSB1*COSB1)
        SC2B2=1.0/(COSB2*COSB2)
        TANB1=SQRTF(SC2B1-1.0)
        TANB2=SQRTF(SC2B2-1.0)
        COST1=(ARD-RNG1*COSB1)/DRG
        COST2=(ARD-RNG2*COSB2)/DRG
        SINT1=SQRTF(1.0-COST1*COST1)
        SINT2=SQRTF(1.0-COST2*COST2)
```

```
      TANB=TANB1
      ITG=1
      CALL CDFGM(QN2,NSIG,TANB,ITG)
      TANB=TANB2
      ITG=2
      CALL CDFGM(QN2,NSIG,TANB,ITG)
      ITRA=1
75    RNNO=RAN1F(-1)
      DO 54 I=1,N
      IF(RNNO-XICD(I))70,71,54
54    CONTINUE
      PRINT 7,XICD(I),RNNO
7     FORMAT(30HOERROR AS MAX VALUE OF XI CDF=E13.6,29H AND RANDOM NUMBE
     1R GENERATED=E15.8)
      STOP
70    J=I-1
      X=(RNNO-XICD(I))*(TANXI(J)-TANXI(I))/(XICD(J)-XICD(I))+TANXI(I)
      GO TO 72
71    X=TANXI(I)
72    GO TO (73,74),ITRA
73    TNXI1=X
      ITRA=2
      GO TO 75
74    TNXI2=X
      ITG=1
81    RNNO=RAN1F(-1)
      DO 57 I=1,NN
      IF(RNNO-GMCD(I,ITG))76,77,57
57    CONTINUE
      PRINT 11,GMCD(I,ITG),RNNO
11    FORMAT(30HOERROR AS MAX VALUE OF GM CDF=E13.6,29H AND RANDOM NUMBE
     1R GENERATED =E15.8)
      STOP
76    J=I-1
      X=(RNNO-GMCD(I,ITG))*(TANGM(J,ITG)-TANGM(I,ITG))/(GMCD(J,ITG)-GMCD
     1(I,ITG)+TANGM(I,ITG)
      GO TO 78
77    GO TO (79,80),ITG
```

```
79     TNGM1=X
       ITG=2
       GO TO 81
80     TNGM2=X
       CALL SUBW(TNXI1,TNXI2,TNGM1,TNGM2,DPHI,TANB1,TANB2,ARD,NP)
       TBMG1=TANB1+TNGM1
       TBMG2=TANB2+TNGM2
       RINV1=1.0/SQRTF(TBMG1*TBMG1+TNXI1*TNXI1*SC2B1)
       RINV2=1.0/SQRTF(TBMG2*TBMG2+TNXI2*TNXI2*SC2B2)
       ACNR1=ABSF((1.0-TANB1*TNGM1)*RINV1)
       ACNR2=ABSF((1.0-TANB2*TNGM2)*RINV2)
       CALL BETA(NS,NP,RNG1,RNG2,R12SQ,ACNR1,ACNR2,TNXI1,TNXI2,RHO1,RHO2,
      1PHI1,PHI2)
       DO 82 I=1,4
       DF=TMUL1*TMUL2*ABR12*COSF(OMEGA*(TIME-TAU))*COSF(OMEGA*(TIME +GSIG
      1-TMU))
       DF2SM(I)=DF2SM(I)+DF*DF
82     F(I)=F(I)+DF
       IF(QNR-QNRLM)87,88,88
87     IF(QNR-PRTLM)86,89,89
88     IPRT=2
89     GCONS=G*37.5/3.1415927
       GCONS=GCONS*GCONS
       DO 90 I=1,4
       GTAV(I)=F(I)*GCONS
90     SSIG(I)=SQRTF(DF2SM(I)-F(I)*F(I)/QNR)/(QNR*QNR-QNR))*0.006332575
       TME=TIME-TO
       IF(LINE-52)102,103,103
103    PRINT 104
104    FORMAT(1H1)
102    PRINT 91,QNR,TME,(GTAV(I),SSIG(I),I=1,4)
91     FORMAT(13HOLOOP NUMBER=F5.0,5X,25HTIME SINCE PULSE ARRIVAL=E13.6/2
      1X,12HAVERAGE G(T),18X,12HAVERAGE G(T),18X,12HAVERAGE G(T),18X,12HA
      2VERAGE G(T)/3X,25HPARAL/PARAL (ERROR SIGMA),4X,10HPARAL/PERP,2X,13
      3H(ERROR SIGMA),4X,10HPERP/PARAL,2X,13H(ERROR SIGMA),5X,9HPERPOPERP
      4,2X,13H(ERROR SIGMA)/1H E13.6,E14.6,E16.6,E14.6,E16.6,E14.6,E16.6,
      5E14.6)
       LINE=LINE+4
```

```
      GO TO(92,93),IPRT
93    TIME=TIME+DT
      IF(TIME-TNPT+GSIG)94,95,95
95    GSIG=GSIG+DSIG
      IF(GSIG-TNPT+TO)96,96,97
97    PRINT 98
98    FORMAT(14HEND OF PROBLEM)
      GO TO 39
      END
C     CALCULATION OF THE VARIABLE W
      SUBROUTINE SUBW(TNXI1,TNXI2,TNGM1,TNGM2,DPHI,TANB1,TANB2,ARD,NP)
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500,2),
     1GMCD(500,2),D2(2),QK(2),ALFA(10),C1(4),SRAV(500),R(500),ABR12(4)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK,COST1,COST2,SINT1,SINT2,ALFA,W,NSIG,SRAV,R,ABR12
      SNDPH=SINF(DPHI)
      CSDPH=COSF(DPHI)
      S=ARD*ACOSF(COST1*COST2+SINT1*SINT2*CSDPH)
      SSUM=0.0
      DO 20 I=1,NP
20    SSUM=SSUM+ALFA(I)*S**I
      RHOCR=EXPF(-SSUM)
      OM2RO=1.0-RHOCR*RHOCR
      COFP=0.0253302956/OM2RO
      COFSM=(TNXI1*TNXI1+TNXI2*TNXI2+TNGM1*TNGM1+TNGM2*TNGM2-2.0*RHOCR*C
     1SDPH*(TNGM1*TNGM2+TNXI1*TNXI2)+2.0*RHOCR*SNDPH*(TNXI2*TNGM1-TNXI1*
     2TNGM2))/(2.0*OM2RO)
      C=0.0
      DO 25 I=1,NSIG
      SIGSQ=SIG(I)*SIG(I)
25    C=C+COFP/(SIGSQ*SIGSQ)*EXPF(-COFSM/SIGSQ)*WT(I)
      Q=C*SQRTF(TNXI1*TNXI2*(TANB1-TNGM1)*(TANB2-TNGM2))
      J=1
      X=TNXI1
      DO 30 K=1,4
      DO 34 I=J,N
      IF(TANXI(I)-X)34,35,36
34    CONTINUE
```

```
      PRINT 1,TANXI(I),X
1     FORMAT(32HOERROR IN W SUBROUTINE AS TANXI(I3,2H)=E13.6,7H AND X=E1
     13.6)
35    J=I
      C=P(I)
      GO TO 37
36    J=I-1
      C=(P(I)-P(J))*(X-TANXI(J))/(TANXI(I)-TANXI(J))+P(J)
37    C1(K)=C
      GO TO (31,32,33,30),K
31    X=TNXI2
      GO TO 30
32    X=TNGM1
      GO TO 30
33    X=TNGM2
30    CONTINUE
      W=Q/(C1(1)*C1(2)*C1(3)*C1(4))
      IF(SENSE SWITCH 5)11,12
11    PRINT 10,TNXI1,TNXI2,TNGM1,TNGM2,DPHI,TANB1,TANB2,ARD,W
10    FORMAT(10HOTAN(XI1)=E13.6,3X,9HTAN(XI2)=E13.6,3X,9HTAN(GM1)=E13.6,
     13X,9HTAN(GM2)=E13.6,3X,10HDELTA PHI=E13.6/12H TAN(BETA1)=E13.6,3X,
     211HTAN(BETA2)=E13.6,3X,7HRADIUS=E13.6,3X,2HW=E15.8)
12    CONTINUE
      RETURN
      END
C     COMPUTATION OF THE CONSTANT K
      SUBROUTINE CONSK(TNXI,ACTNR,RHO1,RHO2,PHI)
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500,2),
     1GMCD(500,2),D2(2),QK(2),ALFA(10),SRAV(500),R(500),ABR12(4)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK,COST1,COST2,SINT1,SINT2,ALFA,W,NSIG,SRAV,R,ABR12
      SN2NR=1.0/(ACTNR*ACTNR+1.0)
      COSNR=SQRTF(1.0-SN2NR)
      XI=ATANF(TNXI)
      PHMXI=PHI-XI
      RADCL=SQRTF(1.0-RHO2*RHO2*SN2NR)
      QK1=(COSNR-RHO1*RADCL)/(COSNR+RHO1*RADCL)
      QK2=(RADCL-RHO1*COSNR)/(RADCL+RHO1*COSNR)
```

```
      COSPX=COSF(PHMXI)
      QK(2)=QK1*QK2*SINF(PHMXI)*COSPX
      COSPX=COSPX*COSPX
      QK(1)=QK1*COSPX+QK2*(1.0-COSPX)
      IF(SENSE SWITCH 3)1,2
1     PRINT 3,TNXI,ACTNR,RHO1,RHO2,PHI,QK(1),QK(2)
3     FORMAT(9HOTAN(XI)=E13.6,3X,27HABSOLUTE VALUE OF COT(N,R)=E13.6,3X,
     17HRHO(1)=E13.6,3X,7HRHO(2)=E13.6/5H PHI=E13.6,3X,12HK(PARALLEL)=E1
     23.6,3X,17HK(PERPENDICULAR)=E13.6)
2     CONTINUE
      RETURN
      END
C     COMPUTATION OF CDF FOR TAN(XI) WITH BIASING
      SUBROUTINE CDFXI(QN1,NSIG)
      DIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500,2),
     1GMCD(500,2),D2(2),QK(2),ALFA(10),SRAV(500),R(500),ABR12(4),F(4),GT
     2AV(4),DF2SM(4),SSIG(4)
      COMMON TANXI,P,XICD,SIG,WT,SO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK,COST1,COST2,SINT1,SINT2,ALFA,W,NSIG,SRAV,R,ABR12
      XO=3.0*SIG(1)
      DELTA=4.0*SQRTF(XO)/QN1
      DX=DELTA*SQRTF(XO)
      X=-XO-DX
      TANXI(1)=X
      P(1)=0.0
      N=1
      Q=0.0
      XICD(1)=0.0
19    N=N+1
      C=0.0
      X=X+DX
      DO 20 I=1,NSIG
      CSIG=SIG(I)
20    C=C+0.3989423*EXPF(-X*X*0.5/(CSIG*CSIG))*WT(I)/CSIG
      Q=Q+C
      DX1=DX
      XICD(N)=Q
      TANXI(N)=X
```

```
      P(N)=C
      DX=DELTA*SQRTF(ABSF(X))
      IF(DX)24,25,26
25    DX=DX1
26    IF(X-XO)19,21,21
24    PRINT 10,DX
10    FORMAT(21HOPROGRAM ERROR AS DX=E11.4)
      STOP
21    P(N)=0.0
      D1=1.0/Q
      DO 30 I=2,N
30    XICD(I)=XICD(I)*D1
      IF(SENSE SWITCH 1)31,32
31    PRINT 1,QN1,NSIG,(TANXI(I),XICD(I),P(I),I=1,N)
1     FORMAT(29H1NUMBER OF TAN(XI) DIVISIONS=F6.1,32H AND THE NUMBER OF
     1INPUT SIGMAS=I5/5X,7HTAN(XI),8X,18HCUMULATIVE DENSITY,6X,19HGENERA
     2L PROBABILITY/19X,20HFUNCTION FOR TAN(XI),7X,16HDENSITY FUNCTION/(
     3E14.6,8X,E13.6,12X,E13.6))
32    CONTINUE
      RETURN
      END
C     COMPUTATION OF CDF FOR TAN(GAMMA) WITH BIASING
      SUBROUTINE CDFGM(QN2,NSIG,TANB,ITG)
      DIMENSION TANXI(500),(,500),XICD(500),SIG(10),WT(10),TANGM(500,2),
     1GMCD(500,2),D2(2),QK(2),ALFA(10),SRAV(500),R(500),ABR12(4)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK,COST1,COST2,SINT1,SINT2,ALFA,W,NSIG,SRAV,R,ABR12
      XOA=3.0*SIG(1)
      SQRTN=SQRTF(TANB+XOA)
      DELTA=2.0*(SQRTF(ABSF(TANB-XOA))+SQRTN)/QN2
      DX=DELTA*SQRTN
      X=-XOA-DX
      NN=1
      GMCD(NN,ITG)=0.0
      Q=0.0
      TANGM(NN,ITG)=X
      J=2
39    X=X+DX
```

```
      DX1=DX
      DX=DELTA*SQRTF(ABSF(TANB+X))
      IF(DX)52,50,51
50    DX1=DX
51    IF(X-TANXI(1))40,40,41
41    IF(X-TANXI(N))42,43,43
42    DO 45 I=J,N
      IF(X-TANXI(I))46,47,45
45    CONTINUE
46    J=J-1
      C=(X-TANXI(J))*(P(I)-P(J))/((TANXI(I)-TANXI(J))+P(J)
      J=I
      GO TO 48
52    PRINT 1,DX
1     FORMAT(34HOPROGRAM ERROR IN GAMMA CDF AS DX=E11.4)
      STOP
40    C=0.0
      GO TO 48
47    J=I
      C=P(I)
48    Q=Q+C
      NN=NN+1
      GMCD(NN,ITG)=Q
      TANGM(NN,ITG)=X
      IF(X-XOA)39,43,43
43    D2(ITG)=1.0/Q
      DO 55 I=1,NN
55    GMCD(I,ITG)=GMCD(I,ITG)*D2
      IF(SENSE SWITCH 2)31,32
31    PRINT 2,QN2,NSIG,TANB,(TANGM(I,ITG),GMCD(I,ITG),I=1,NN)
2     FORMAT(32H1NUMBER OF TAN(GAMMA) DIVISIONS=F6.1,25H, NUMBER OF INPU
     1T SIGMAS=I5,15H AND TAN(BETA)=E13.6/3X,12HTAN(GAMMA,I),7X,18HCUMUL
     2ATIVE DENSITY/20X,23HFUNCTION FOR TAN(GAMMA,I)/(E14.6,10X,E13.6))
32    CONTINUE
      RETURN
      END
C     CALCULATION OF THE VARIABLE BETA(R)*BETA(R+DR) AVERAGE
      SUBROUTINE BETA(NS,NP,RNG1,RNG2,R12SQ,ACNR1,ACNR2,TNXI1,TNXI2,RHO1
```

```
     1,RHO2,PHI1,PHI2)
      SIMENSION TANXI(500),P(500),XICD(500),SIG(10),WT(10),TANGM(500,2),
     1GMCD(500,2),D2(2),QK(2),ALFA(10),SRAV(500),R(500),ABR12(4)
      COMMON TANXI,P,XICD,SIG,WT,XO,DX,X,C,Q,DX1,N,DELTA,NN,TANGM,GMCD,D
     11,D2,QK,COST1,COST2,SINT1,SINT2,ALFA,W,NSIG,SRAV,R,ABR12
      DO 20 I=1,NS
      IF(R(I)-RNG1)20,21,22
20    CONTINUE
      PRINT 1,RNG1,R(I)
1     FORMAT(34HOERROR IN SUBROUTINE BETA AS RNG1=E15.8,11H AND R(NS)=E1
     15.8)
      STOP
22    J=I-1
      SRAV1=(RNG1-R(J))*(SRAV(I)-SRAV(J))/(R(I)-R(J))+SRAV(J)
      GO TO 23
21    SRAV1=SRAV(I)
23    DO 24 I=1,NS
      IF(R(I)-RNG2)24,25,26
24    CONTINUE
      PRINT 2,RNG2,R(I)
2     FORMAT(34HOERROR IN SUBROUTINE BETA AS RNG2=E15.8,11H AND R(NS)=E1
     15.8)
      STOP
26    J=I-1
      SRAV2=(RNG2-R(J))*(SRAV(I)-SRAV(J))/(R(I)-R(J))+SRAV(J)
      GO TO 27
25    SRAV2=SRAV(I)
27    SAV12=SRAV1*SRAV2
      BCONS=D1*D2(1)*D2(2)*SAV12*ACNR1*ACNR2/R12SQ
      TNXI=TNXI1
      ACTNR=ACNR1
      PHI=PHI1
      CALLCONSK(TNXI,ACTNR,RHO1,RHO2,PHI)
      QK11=QK(1)
      QK12=QK(2)
      ACTNR=ACNR2
      PHI=PHI2
      TNXI=TNXI2
```

```
      CALL CONSK(TNXI,ACTNR,RHO1,RHO2,PHI)
      ABR12(1)=BCONS*QK11*QK(1)
      ABR12(2)=BCONS*QK11*QK(2)
      ABR12(3)=BCONS*QK12*QK(1)
      ABR12(4)=BCONS*QK12*QK(2)
      IF(SENSE SWITCH 4)11,12
11    PRINT 10,RNG1,RNG2,ACNR1,TNXI1,ACNR2,TNXI2,RHO1,RHO2,PHI1,PHI2,(AB
     1R12(I),I=1,4)
10    FORMAT(10HORANGE(1)=E13.6,3X,9HRANGE(2)=E13.6,3X,28HABSOLUTE VALUE
     1 VALUE OF COT(N,R1)=E13.6,3X,9HTAN(XI1)=E13.6/29H ABSOLUTE VALUE O
     2F COT(N,R2)=E13.6,3X,9HTAN(XI2)=E13.6,3X,7HRHO(1)=E13.6,3X,7HRHO(2
     3)=E13.6/8H PHI(1)=E13.6,3X,7HPHI(2)=E13.6,3X,21HBETA AV(PARAL/PARA
     4L)=E13.6,3X,20HBETA AV(PARAL/PERP)=E13.6/21H BETA AV(PERP/PARAL)=E
     513.6,3X,19HBETA AV(PERP/PERP)=E13.6)
12    CONTINUE
      RETURN
      END
```